On the Capacity of Interference Channels with a Partially-Cognitive Transmitter

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***Abstract*— An achievable region, outer bounds and a capacity result are established for two-sender two-receiver interference channels with one cognitive transmitter. Specifically, we assume that one transmitter knows either the full or, more realistically, the partial message of the other transmitter due to its cognitive capabilities. The achievable region is obtained by a rate-splitting strategy, which generalizes prior strategies under both weak and strong interference conditions. The outer bounds are based on an extension of the Nair-El Gamal outer bound for the broadcast channel capacity. When only the partial message is known to the cognitive user, the capacity region in strong interference is established. In this regime, the interference is such that both receivers can decode both messages with no rate penalty.**

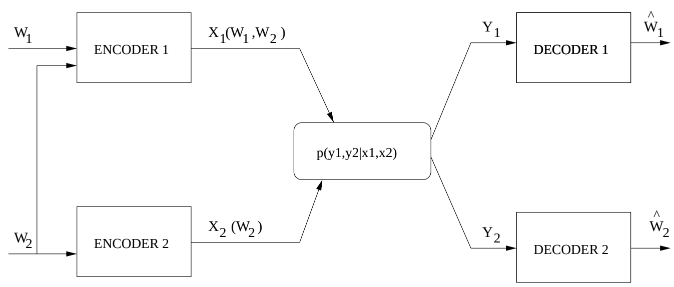

Fig. 1. Interference channel with unidirectional cooperation.

I. Introduction and Related Work

Two-sender, two-receiver channel models allow for various forms of transmitter cooperation. An encoder that has knowledge about the other user's message can use it to improve its own rate and the other user's rate. The level of cooperation and performance improvement will depend on the amount of information the encoders share. When senders are unaware of each other's messages, we have the interference channel [1], [2]. This paper considers channel models in which one sender knows either the full message of the other user, allowing for *full unidirectional* cooperation, or a part of the message allowing for *partial unidirectional* cooperation.

The considered channel models have some characteristics of networks with cognitive users. Cognitive radio [3] technology is aimed at developing smart radios that are both aware of and adaptive to the environment. Such radios can efficiently sense the spectrum, decode information from detected signals and use that knowledge to improve the system performance. This technology motivates new information-theoretic models that try to capture the cognitive radio characteristics. Somewhat idealistically, we assume that if a user is cognitive, it knows either the full message or, more realistically, a part of the message of the other encoder. The interference channel with full unidirectional cooperation was dubbed the *cognitive radio channel* and achievable rates were presented in [4], [5]. The capacity region for the Gaussian case of weak interference was determined in [6] and [7]. A more general scheme was proposed in [8]. We present a scheme that generalizes the ones in [6]-[9]. Our scheme is similar to the one in [8]: as in [8] and [4], the encoders use rate-splitting [2] to enable the receivers to decode part of the interference; the cognitive transmitter cooperates in sending the other user's message and uses Gel'fand-Pinsker binning to reduce interference to its receiver. The key difference of our contribution is in the way the binning is performed. We also use the ideas of [10] and [11] that, respectively, extend [12] and [13] to channels with different states non-causally known to the encoder.

The assumption that the full message of one user is available to the cognitive user may be an over-idealized model of the cognitive network. Its capacity constitutes an outer bound on the performance of more realistic models. For that reason, we also consider a more general model in which only a part of the message is known to the cognitive user. The proposed achievable strategy generalizes to this case. We present outer bounds for this case that extend the Nair-El Gamal broadcast outer bound, [14]. For the full unidirectional cooperation, a similar bound was presented in [9]. We also consider the strong interference scenario in which interference is such that both decoders can decode both messages with no penalty. The capacity region of the strong interference channel with full unidirectional cooperation was determined in [15], [16]. By applying the same approach, we obtain the capacity region of the interference channel with partial cooperation in strong interference.

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II. The Interference Channel with Full Unidirectional Cooperation

Consider a channel with finite input alphabets $\mathcal{X}_1, \mathcal{X}_2$, finite output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$, and a conditional probability distribution $p(y_1, y_2|x_1, x_2)$, where $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ are channel outputs. Each encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. Message W_2 is also known at encoder 1, thus allowing for full unidirectional cooperation (see Fig. 1). The channel is memoryless and time-invariant in the sense that

$$\begin{aligned} &p(y_{1,n}, y_{2,n}|x_1^n, x_2^n, y_1^{n-1}, y_2^{n-1}, \bar{w}) \\ &\quad = p_{Y_1,Y_2|X_1,X_2}(y_{1,n}, y_{2,n}|x_{1,n}, x_{2,n}) \end{aligned} \tag{1}$$

for all n, where X_1, X_2 and Y_1, Y_2 are random variables representing the respective inputs and outputs, $\bar{w} = [w_1, w_2]$ denotes the messages to be sent, and $x_t^n = \begin{bmatrix} x_{t,1}, & \ldots, & x_{t,n} \end{bmatrix}$. We will follow the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables.

An (M_1, M_2, N, P_e) code has two encoding functions

$$X_1^N = f_1(W_1, W_2) \tag{2}$$

$$X_2^N = f_2(W_2) \tag{3}$$

two decoding functions

$$\hat{W}_t = g_t(Y_t^N) \quad t = 1, 2 \tag{4}$$

and an error probability

$$P_e = \max\{P_{e,1}, P_{e,2}\} \tag{5}$$

where, for $t = 1, 2$, we have

$$P_{e,t} = \sum_{(w_1,w_2)} \frac{1}{M_1 M_2} P\left[g_t(Y_t^N) \neq w_t|(w_1, w_2) \text{ sent}\right]. \tag{6}$$

A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_1, M_2, N, P_e) code such that

$$M_t \geq 2^{NR_t}, \quad t = 1, 2, \text{ and } P_e \leq \epsilon.$$

The capacity region of the interference channel with full unidirectional cooperation is the closure of the set of all achievable rate pairs (R_1, R_2).

A. Inner Bound

To obtain an inner bound, we employ rate splitting. We let

$$R_1 = R_{1a} + R_c \tag{7}$$

$$R_2 = R_{2a} + R_{2b} \tag{8}$$

for nonnegative $R_{1a}, R_c, R_{2a}, R_{2b}$ which we now specify.

In the encoding scheme, encoder 2 uses superposition coding with two codebooks X_{2a}^N, X_{2b}^N. Encoder 1 repeats the steps of encoder 2 and adds binning: it encodes the split message W_1 with two codebooks which are Gel'fand-Pinsker precoded against X_{2a}^N, X_{2b}^N. In particular:

1) Binning against X_{2a}^N, X_{2b}^N is used to create a codebook U_{1c}^N of common rate R_c.
2) Binning against X_{2a}^N, X_{2b}^N conditioned on U_{1c} is used to create a codebook U_{1a}^N with private rate R_{1a}.

The encoding structure is shown in Fig. 2.

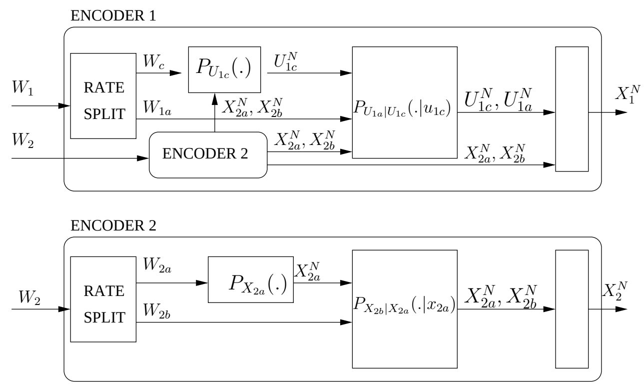

Fig. 2. Encoding structure.

For the interference channel with full unidirectional cooperation we have the following result.

Theorem 1: (sequential decoding) Rates (7)-(8) are achievable if

$$R_{1a} \leq I(U_{1a}; Y_1|U_{1c}, Q) - I(U_{1a}; X_{2a}, X_{2b}|U_{1c}, Q) \tag{9}$$

$$\begin{aligned} R_c \leq &\min\{I(U_{1c}; Y_1|Q), I(U_{1c}; Y_2, X_{2a}|Q)\} \\ &- I(U_{1c}; X_{2a}, X_{2b}|Q) \end{aligned} \tag{10}$$

$$R_{2a} \leq I(X_{2a}; Y_2|Q) \tag{11}$$

$$R_{2b} \leq I(X_{2b}; Y_2, U_{1c}|X_{2a}, Q) \tag{12}$$

for some joint distribution that factors as $p(q)p(x_{2a}, x_{2b}, u_{1c}, u_{1a}, x_1, x_2|q)p(y_1, y_2|x_1, x_2)$ and for which the right-hand side of (9) and (10) are nonnegative. Q is a time sharing random variable.

Proof: See the appendix. ■

Theorem 2: (joint decoding) Rates (7)-(8) are achievable if

$$\begin{aligned} R_{1a} \leq &I(U_{1a}; Y_1|U_{1c}, Q) \\ &- I(U_{1a}; X_{2a}, X_{2b}|U_{1c}, Q) \end{aligned} \tag{13}$$

$$\begin{aligned} R_{1a} + R_c \leq &I(U_{1c}, U_{1a}; Y_1|Q) \\ &- I(U_{1c}, U_{1a}; X_{2a}, X_{2b}|Q) \end{aligned} \tag{14}$$

$$R_{2a} + R_{2b} \leq I(X_{2a}, X_{2b}; Y_2, U_{1c}|Q) \tag{15}$$

$$R_{2a} + R_{2b} + R_c \leq I(X_{2a}, X_{2b}, U_{1c}; Y_2|Q) \tag{16}$$

$$R_{2b} \leq I(X_{2b}; Y_2, U_{1c}|X_{2a}, Q) \tag{17}$$

$$R_{2b} + R_c \leq I(X_{2b}, U_{1c}; Y_2|X_{2a}, Q) \tag{18}$$

for some joint distribution that factors as $p(q)p(x_{2a}, x_{2b}, u_{1c}, u_{1a}, x_1, x_2|q)p(y_1, y_2|x_1, x_2)$ and for which all the right-hand sides are nonnegative.

Proof: An outline is given in the appendix. ■

Remark: The rates of Thm. 2 include the rates of Thm. 1. Theorem 2 also includes the rates of the following schemes:

- The scheme of [6, Thm 3.1] for $X_{2a} = \emptyset, U_{1c} = \emptyset, X_{2b} = (X_2, U)$ and $U_{1a} = V$ achieving:

$$R_2 \leq I(X_2, U; Y_2) \tag{19}$$

$$R_1 \leq I(V; Y_1) - I(V; X_2, U) \tag{20}$$

for $p(u,x_2)p(v|u,x_2)p(x_1|v)$.

- The scheme of [17, Lemma 4.2] for $X_{2a}=\emptyset$, $X_{2b}=X_2$, $U_{1a}=\emptyset$, and $R_1=R_c$, $R_2=R_{2b}$ as:

$$R_2 \le I(X_2;Y_2|U_{1c})$$
$$R_1 \le \min\{I(U_{1c};Y_1), I(U_{1c};Y_2)\}$$

for $p(x_2)p(u_{1c})$. The strategy in [17] considers the case when $I(U_{1c};Y_1) \le I(U_{1c};Y_2)$.

- Carbon-copy on dirty paper [11] for $X_{2a}=\emptyset, U_{1a}=\emptyset$.
- For $X_{2a}=\emptyset$, our scheme closely resembles the scheme in [8]. One difference in our scheme is that two binning steps are not done independently which brings potential improvements.

It is also interesting to compare our scheme to the encoding scheme in [4]. The latter combines rate splitting at both users, with two-step binning at the cognitive user. Each user sends a private index decoded by its receiver, and a common index decoded by both. Again, one difference in our scheme is that two binning steps are not independent.

III. The Interference Channel with Partial Unidirectional Cooperation

We next assume that the cognitive user knows only a partial message of the other user. We model this by letting encoder 2 send two messages, W_0, W_2, to receiver 2 where only W_0 is known to encoder 1. As before, transmitter 1 also sends W_1 to receiver 1. The two encoding functions become

$$X_1^N = f_1(W_1, W_0) \tag{21}$$
$$X_2^N = f_2(W_2, W_0) \tag{22}$$

and the decoding functions become

$$\hat{W}_1 = g_1(Y_1^N) \tag{23}$$
$$(\hat{W}_0, \hat{W}_2) = g_2(Y_2^N) \tag{24}$$

We refer to this channel as the interference channel with *partial unidirectional cooperation*. We are interested in achievable rate triples (R_0, R_1, R_2).

A. Outer Bound

Theorem 3: The set of rate triples (R_0,R_1,R_2) satisfying

$$R_0 \le I(V;Y_2) \tag{25}$$
$$R_1 \le I(V,U_1;Y_1) \tag{26}$$
$$R_0+R_2 \le I(V,U_2;Y_2) \tag{27}$$
$$R_1+R_2 \le I(V,U_1;Y_1)+I(U_2;Y_2|V,U_1) \tag{28}$$
$$R_0+R_1+R_2 \le I(U_1;Y_1|U_2,V)+I(V,U_2;Y_2) \tag{29}$$

for input distributions $p(v,u_1,u_2,x_1,x_2)$ that factor as

$$p(u_1)p(u_2)p(v|u_1,u_2)p(x_2|v,u_2)p(x_1|v,u_1,u_2,x_2) \tag{30}$$

is an outer bound to the capacity region of the interference channel with partial unidirectional cooperation.
We also have the following result:

Theorem 4: The set of rate triples (R_0,R_1,R_2) satisfying

$$R_0 \le I(V,U_0;Y_2) \tag{31}$$
$$R_1 \le I(V,U_0,U_1;Y_1) \tag{32}$$
$$R_0+R_2 \le I(V,U_0,U_2;Y_2) \tag{33}$$
$$R_1+R_2 \le I(V,U_0,U_1;Y_1)+I(U_2;Y_2|V,U_0,U_1) \tag{34}$$
$$R_0+R_1+R_2 \le I(U_1;Y_1|V,U_0,U_2)+I(V,U_0,U_2;Y_2) \tag{35}$$
$$R_0+R_1+R_2 \le I(U_1,V;Y_1)+I(U_0,U_2;Y_2|U_1,V) \tag{36}$$

for input distributions $p(v,u_0,u_1,u_2,x_1,x_2)$ that factor as

$$p(u_0)p(u_1)p(u_2)p(v|u_0,u_1,u_2) \; p(x_2|v,u_0,u_2)p(x_1|v,u_0,u_1,u_2,x_2) \tag{37}$$

is an outer bound to the capacity region of the interference channel with partial unidirectional cooperation.
Setting $R_2=0$, $U_2=\emptyset$ in Thm. 3, and redefining R_0 as R_2 yields an outer bound to the capacity region of the interference channel with full unidirectional cooperation.

B. Capacity Region in Strong Interference

The approach of [15], [16] can be applied to determine the capacity region in strong interference.

Theorem 5: An interference channel with partial unidirectional cooperation that satisfies the strong interference conditions [18]

$$I(X_1;Y_1|X_2) \le I(X_1;Y_2|X_2) \tag{38}$$
$$I(X_2;Y_2|X_1) \le I(X_2;Y_1|X_1) \tag{39}$$

for input distributions $p(x_1,x_2)$ that factor as $p(x_1)p(x_2)$ and

$$I(X_1,X_2;Y_2) \le I(X_1,X_2;Y_1) \tag{40}$$

for all $p(x_1,x_2)$, has the capacity region

$$\mathcal{C} = \bigcup \{(R_0,R_1,R_2) : R_0 \ge 0, R_1 \ge 0, R_2 \ge 0$$
$$R_1 \le I(X_1;Y_1|X_2,U) \tag{41}$$
$$R_2 \le I(X_2;Y_2|X_1,U) \tag{42}$$
$$R_1+R_2 \le \min\{I(X_1,X_2;Y_1|U), I(X_1,X_2;Y_2|U)\} \tag{43}$$
$$R_0+R_1+R_2 \le I(X_1,X_2;Y_2) \tag{44}$$

where the union is over all joint distributions that factor as

$$p(u)p(x_1|u)p(x_2|u)p(y_1,y_2|x_1,x_2). \tag{45}$$

Proof: See the appendix. ■

IV. Conclusion

We developed an encoding strategy for the interference channel with full unidirectional cooperation that generalizes previously proposed encoding strategies. We plan to further evaluate its performance and compare it to the performance of other schemes, focusing on the Gaussian channel. For the interference channel with partial unidirectional cooperation, we developed a new outer bound that extends the Nair-El Gamal broadcast outer bound, and obtained the capacity result in the strong interference.

	Error event	Arbitrarily small positive error probability if
E_1	$(\hat{w}_c \neq 1, w_{1a} = 1)$	$R_c + R'_c \leq I(U_{1c}, U_{1a}; Y_1)$
E_2	$(\hat{w}_c = 1, w_{1a} \neq 1)$	$R_{1a} + R'_{1a} \leq I(U_{1a}; Y_1 \| U_{1c})$
E_3	$(\hat{w}_c \neq 1, w_{1a} \neq 1)$	$R_c + R'_c + R_{1a} + R'_{1a} \leq I(U_{1c}, U_{1a}; Y_1)$
E'_1	$(\hat{w}'_{2a} \neq 1, w'_{2b} = 1, w'_c = 1)$	$R_{2a} \leq I(X_{2a}, X_{2b}; Y_2, U_{1c})$
E'_2	$(\hat{w}'_{2a} \neq 1, w'_{2b} \neq 1, w'_c = 1)$	$R_{2a} + R_{2b} \leq I(X_{2a}, X_{2b}; Y_2, U_{1c})$
E'_3	$(\hat{w}'_{2a} \neq 1, w'_{2b} = 1, w'_c \neq 1)$	$R_{2a} + R_c + R'_c \leq I(X_{2a}, X_{2b}, U_{1c}; Y_2) + I(U_{1c}; X_{2a}, X_{2b})$
E'_4	$(\hat{w}'_{2a} \neq 1, w'_{2b} \neq 1, w'_c \neq 1)$	$R_{2a} + R_{2b} + R_c + R'_c \leq I(X_{2a}, X_{2b}, U_{1c}; Y_2) + I(U_{1c}; X_{2a}, X_{2b})$
E'_5	$(\hat{w}'_{2a} = 1, w'_{2b} \neq 1, w'_c = 1)$	$R_{2b} \leq I(X_{2b}; Y_2, U_{1c} \| X_{2a})$
E'_6	$(\hat{w}'_{2a} = 1, w'_{2b} \neq 1, w'_c \neq 1)$	$R_{2b} + R_c + R'_c \leq I(X_{2b}, U_{1c}; Y_2 \| X_{2a}) + I(U_{1c}; X_{2a}, X_{2b})$

TABLE I

ERROR EVENTS IN JOINT DECODING AND CORRESPONDING RATE BOUNDS.

V. APPENDIX

Proof: (Theorem 1) **Code construction:** Ignore Q. Choose a distribution $p(x_{2a}, x_{2b}, u_{1c}, u_{1a}, x_1, x_2)$.

- Split the rates as in (7)-(8).
- Generate $2^{NR_{2a}}$ codewords $x_{2a}^N(w_{2a})$ using $P_{X_{2a}}(\cdot)$, $w_{2a} = 1, \ldots, 2^{NR_{2a}}$.
- For each w_{2a}: Generate $2^{NR_{2b}}$ codewords $x_{2b}^N(w_{2a}, w_{2b})$ using $P_{X_{2b}|X_{2a}}(\cdot|x_{2a})$, $w_{2b} = 1, \ldots, 2^{NR_{2b}}$, where $x_{2a} = x_{2a,i}(w_{2a})$. Similar notation is used in the rest of the code construction.
- For each pair (w_{2a}, w_{2b}) : Generate $x_2^N(w_{2a}, w_{2b})$. It can be shown that it is enough to choose x_2 to be a deterministic function of (x_{2a}, x_{2b}).
- Generate $2^{N(R_{1c}+R_{1c'})}$ codewords $u_{1c}^N(w_c, b_c)$, $w_c = 1, \ldots, 2^{NR_{1c}}$, $b_c = 1, \ldots, 2^{NR_{1c'}}$ using $P_{U_{1c}}(\cdot)$.
- For each $u_{1c}^N(w_c, b_c)$: Generate $2^{N(R_{1a}+R'_{1a})}$ codewords $u_{1a}^N(w_c, b_c, w_{1a}, b_{1a})$, $w_{1a} = 1, \ldots, 2^{NR_{1a}}$, $b_{1a} = 1, \ldots, 2^{NR'_{1a}}$ using $P_{U_{1a}|U_{1c}}(\cdot|u_{1c})$.
- For (w_1, w_2) : Generate $x_1^N(w_{2a}, w_{2b}, w_c, b_c, w_{1a}, b_{1a})$ where x_1 is a deterministic function of $(x_{2a}, x_{2b}, u_{1c}, u_{1a}, x_2)$.

Encoders: Encoder 1:

1) Split the NR_1 bits w_1 into NR_{1a} bits w_{1a} and NR_c bits w_c. Similarly, split the NR_2 bits w_2 into NR_{2a} bits w_{2a} and NR_{2b} bits w_{2b}. We write this as
$$w_1 = (w_{1a}, w_c), \qquad w_2 = (w_{2a}, w_{2b}).$$
2) Try to find a bin index b_c so that $(u_{1c}^N(w_c, b_c), x_{2a}^N(w_{2a}), x_{2b}^N(w_{2a}, w_{2b})) \in T_\epsilon(P_{U_{1c}X_{2a}X_{2b}})$. If no such b_c is found, choose $b_c = 1$.
3) For each (w_c, b_c): Try to find a bin index b_{1a} such that $(u_{1a}^N(w_c, b_c, w_{1a}, b_{1a}), x_{2a}^N(w_{2a}), x_{2b}^N(w_{2a}, w_{2b}), u_{1c}^N(w_c, b_c)) \in T_\epsilon(P_{U_{1a}X_{2a}X_{2b}U_{1c}})$. If cannot, choose $b_{1a} = 1$.
4) Transmit x_1^N.

Encoder 2: Transmit x_2^N.

Decoders: Decoder 1: Given y_1^N:

1) Choose $(\hat{w}_c, \hat{b}_c)$ if $(u_{1c}^N(\hat{w}_c, \hat{b}_c), y_1^N) \in T_\epsilon(P_{U_{1c}Y_1})$.
2) Choose $(\hat{w}_{1a}, \hat{b}_{1a})$ if $(u_{1a}^N(\hat{w}_c, \hat{b}_c, \hat{w}_{1a}, \hat{b}_{1a}), u_{1c}^N(\hat{w}_c, \hat{b}_c), y_1^N) \in T_\epsilon(P_{U_{1a}U_{1c}Y_1})$.

When there are multiple pairs that satisfy one of the above conditions, choose one pair.

Decoder 2: Given y_2^N:

1) Choose $\hat{w}'_{2a}$ if $(x_{2a}^N(\hat{w}'_{2a}), y_2^N) \in T_\epsilon(P_{X_{2a}Y_2})$.
2) Choose $(\hat{w}'_c, \hat{b}'_c)$ if $(u_{1c}^N(\hat{w}'_c, \hat{b}'_c), x_{2a}^N(\hat{w}'_{2a}), y_2^N) \in T_\epsilon(P_{U_{1c}X_{2a}Y_2})$.
3) Choose $\hat{w}'_{2b}$ if $(x_{2b}^N(\hat{w}'_{2a}, \hat{w}'_{2b}), u_{1c}^N(\tilde{w}'_c, \tilde{b}'_c), x_{2a}^N(\hat{w}'_{2a}), y_2^N) \in T_\epsilon(P_{X_{2b}U_{1c}X_{2a}Y_2})$.

If no message can be chosen in any step, declare an error.

Analysis: See [9]. ■

Proof: (Theorem 2) The code construction and encoders are the same as in the proof of Thm. 1.

Decoders: Decoder 1: Given y_1^N, choose $(\hat{w}_c, \hat{b}_c, \hat{w}_{1a}, \hat{b}_{1a})$ if $(u_{1c}^N(\hat{w}_c, \hat{b}_c), u_{1a}^N(\hat{w}_c, \hat{b}_c, \hat{w}_{1a}, \hat{b}_{1a}), y_1^N) \in T_\epsilon(P_{U_{1c}U_{1a}Y_1})$. If there is more than one such a quadruple, choose one.

Decoder 2: Given y_2^N, choose $(\hat{w}'_{2a}, \hat{w}'_c, \hat{b}'_c, \hat{w}'_{2b})$ if $(x_{2a}^N(\hat{w}'_{2a}), u_{1c}^N(\hat{w}'_c, \hat{b}'_c), x_{2b}^N(\hat{w}'_{2a}, \hat{w}'_{2b}), y_2^N) \in T_\epsilon(P_{X_{2a}U_{1c}X_{2b}Y_2})$. If there is more than one such a quadruple, choose one.

Analysis: Table I shows the possible error events and the corresponding rate bounds that guarantee that the error probability of each event can be made small as N gets large. The other bounds for events E_1, E'_1, E'_3 are loose. The other rate expressions in Table I yield (13)-(18). ■

Proof (Theorem 3): Consider an (M_0, M_1, M_2, N, P_e) code. We start by deriving (27); (25) and (26) follow by similar steps. Fano's inequality implies that for reliable communication we require

$$N(R_0 + R_2) \leq I(W_0, W_2; Y_2^N) \tag{46}$$
$$\leq \sum_{i=1}^{N} I(W_0, W_2, Y_1^{i-1}, Y_{2,i+1}^N; Y_{2,i}) \tag{47}$$
$$= \sum_{i=1}^{N} I(U_{2,i}, V_i; Y_{2,i}) \tag{48}$$

where $Y_{t,i}^j = (Y_{t,i}, \ldots, Y_{t,j})$. To obtain (48) we introduce, for $i = 1, \ldots, N$, auxiliary random variables

$$V_i = (W_0, Y_1^{i-1}, Y_{2,i+1}^N), \quad U_{1,i} = W_1, \quad U_{2,i} = W_2. \tag{49}$$

Similarly, choosing $U_{0,i} = W_0$, $V_i = (Y_1^{i-1}, Y_{2,i+1}^N)$ yields a bound that corresponds to (33)

$$N(R_0 + R_2) \le \sum_{i=1}^N I(V_i, U_{0,i}, U_{2,i}; Y_{2,i}). \qquad (50)$$

We next consider the bound (29). Fano's inequality implies that for reliable communication we require

$$
\begin{aligned}
&N(R_0 + R_1 + R_2) && (51)\\
&\le I(W_1; Y_1^N) + I(W_0, W_2; Y_2^N) && (52)\\
&\le I(W_1; Y_1^N | W_0, W_2) + I(W_0, W_2; Y_2^N) && (53)\\
&= \sum_{i=1}^N I(W_1; Y_1^i | W_0, W_2, Y_{2,i+1}^N) - I(W_1; Y_1^{i-1} | W_0, W_2, Y_{2,i}^N) \\
&\quad + I(W_0, W_2; Y_{2,i} | Y_{2,i+1}^N) && (54)\\
&= \sum_{i=1}^N I(W_1; Y_1^i | W_0, W_2, Y_{2,i+1}^N) \\
&\quad - [I(W_1, Y_{2,i}; Y_1^{i-1} | W_0, W_2, Y_{2,i+1}^N) \\
&\quad - I(Y_{2,i}; Y_1^{i-1} | W_0, W_2, Y_{2,i+1}^N)] + I(W_0, W_2; Y_{2,i} | Y_{2,i+1}^N) \\
&= \sum_{i=1}^N I(W_1; Y_{1,i} | W_2, V_i) - I(Y_{2,i}; Y_1^{i-1} | W_1, W_0, W_2, Y_{2,i+1}^N) \\
&\quad + I(W_0, W_2, Y_1^{i-1}; Y_{2,i} | Y_{2,i+1}^N) && (55)\\
&\le \sum_{i=1}^N I(U_{1,i}; Y_{1,i} | U_{2,i}, V_i) + I(U_{2,i}, V_i; Y_{2,i}) && (56)
\end{aligned}
$$

where (53) follows from the independence of W_0, W_1, W_2; (54) follows by expanding the first and second mutual information expressions in (53) as a sum of differences and using the chain rule for mutual information, respectively; (55) follows by using the chain rule for mutual information; and (56) follows by using the non-negativity of mutual information.

Note that a bound symmetric to (29) in which roles of (U_1, Y_1) and (U_2, Y_2) are interchanged cannot be established because W_0 is not required at decoder 1 and hence an inequality symmetric to (53) may not hold. Instead, following similar steps as above, we derive (28) as:

$$
\begin{aligned}
&N(R_1 + R_2) && (57)\\
&\le I(W_1; Y_1^N) + I(W_2; Y_2^N) && (58)\\
&\le I(W_0, W_1; Y_1^N) + I(W_2; Y_2^N | W_0, W_1) && (59)
\end{aligned}
$$

Note that (59) is symmetric to (53). The steps (53)-(56) can therefore be applied to show that

$$N(R_1 + R_2) \le \sum_{i=1}^N I(U_{2,i}; Y_{2,i} | U_{1,i}, V_i) + I(U_{1,i}, V_i; Y_{1,i}).$$

Following standard methods, as in [14], the obtained bounds can be reduced to their single-letter characterizations. We observe from (49) that $U_{1,i}$ and $U_{2,i}$ are independent. Furthermore, due to unidirectional cooperation, the following is a Markov chain

$$U_1 \to (V, U_2) \to X_2. \qquad (60)$$

Hence, the joint probability distribution factors as in (30). □

Proof (Theorem 4): Bounds (31)-(35) follow directly from Thm. 3 by redefining V to be (V, U_0). Bound (36) can be proved by following the same steps as the Thm. 3 proof. □

Proof: (Theorem 5) Under conditions (38)-(40), the rates (41)-(44) are the capacity region of the *compound multiaccess channel* where (W_0, W_1, W_2) are required at both decoders [15]. When such decoding constraints are relaxed, as in the considered channel, the rates (41)-(44) are still achievable. For the converse, (41) and (42) follow by standard methods. Following the steps in [15], one can derive (43) and (44). ■

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