VECTOR MAC CAPACITY REGION WITH COVARIANCE FEEDBACK

Technical Area : Multiuser Information Theory

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Abstract

We analyze the capacity region for the multiple access channel with multiple transmit and receive antennas. For Rayleigh Fading with perfect channel state information at the receiver (CSIR) and only a knowledge of the spatial correlations at the transmitter, we determine the optimal transmit strategies to achieve the points on the boundary of the capacity region under various correlation models applicable to different practical systems. We also explore the geometrical shape of the capacity region as a function of the channel correlations.

I. Introduction

The capacity of multiple antenna systems with partial channel state information at the transmitter (CSIT) and perfect CSIR has been the subject of much research recently. For a single user, solutions to capacity optimization, optimality of beamforming (scalar coding with transmit precoding), and average SNR maximization have been explored in [1], [2], [3] and [4]. However capacity results with multiple users and partial side information are practically nonexistent. In this work we analyze the capacity region and determine the optimal transmit strategies to achieve the points on the boundary of the capacity region under various correlation models applicable to different practical systems.

II. System Model

A K user MIMO multiple access channel with the i^{th} user having n_{T_i} transmit antennas and with n_R receive antennas (denoted as an $(\mathbf{n_T}, n_R)$ channel) has received signal given by

$$\mathbf{y}(k) = \sum_{i=1}^{K} \mathbf{H_i}(k)\mathbf{x_i}(k) + \mathbf{n}(k), \tag{1}$$

where at time instant k, $\mathbf{y}(k)$ is the n_R dimensional received vector, $\mathbf{x_i}(k)$ is the n_{T_i} dimensional input vector from user i, $\mathbf{n}(k)$ is the n_R dimensional white Gaussian noise vector, and $\mathbf{H_i}(k)$ describes the i^{th} user's channel matrix. $\mathbf{n_T}$ is a K dimensional vector with i^{th} element given by n_{T_i}. For convenience we drop the time index k and normalize the noise variance to unity. At each transmitter the elements of $\mathbf{H_i}$ are modeled as complex Gaussian random variables. While the elements of $\mathbf{H_i}$ and $\mathbf{H_j}$ are uncorrelated for $i \neq j$, the elements within each channel matrix $\mathbf{H_i}$ can be correlated within themselves. Each transmitter possesses full information about all the channel correlations for its own channel as well as for every other user's channel. Notationally, define the vector $\tilde{\mathbf{H}} \triangleq \text{vec}([\mathbf{H_1}\ \mathbf{H_2}\ \cdots\ \mathbf{H_K}])$, where, similar to the notation in [9], for a matrix A, $\text{vec}(A)$ is used to denote the vector obtained by stacking the columns of A under each other. Further define the block diagonal covariance matrix $\Sigma \triangleq \mathrm{E}[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\dagger}]$. So our assumption is that Σ is known to each transmitter.

The channel correlations either do not change with time, or they change at a much slower rate than the channel itself and can be evaluated at the receiver and fed back to each transmitter periodically through a low rate reliable feedback channel. The delay on the feedback channel is assumed to be negligible compared to the duration for which the channel correlations are assumed fixed. Also, since the channel correlations change at a very slow rate, we do not define a stochastic model for the correlations themselves. Instead we are interested in the multiple access MIMO capacity region as a function of given fixed channel correlations.

Based on the capacity region characterization for a constant vector multiple access channel in [5], the single antenna multiple access capacity region with no CSIT derived in [6] and the intuitive idea in [7], the capacity region of our system can be expressed as

$$C(\mathbf{P},\Sigma) \equiv \mathrm{co}_{\{\mathbf{Q}:\mathrm{trace}[Q_j]\leq P_j, 1\leq j\leq K\}} \left(\left\{ \mathbf{R} : \sum_{i\in S} R_i \leq \mathbb{E}_{\mathbf{H}}\left[\frac{1}{2}\log\left| \sum_{i\in S} \mathbf{H_i}\mathbf{Q_i}\mathbf{H_i^{\dagger}} + \mathbf{I} \right| \right] \right\} \right) \quad \forall \; S \subset \{1,2,\ldots,K\}, \tag{2}$$

where $\mathbf{P}$ is the vector of the users' transmit power constraints, $|A|$ is the determinant of the matrix A and $\mathrm{co}(S)$ is the convex hull of the points in the set S. The required input distribution for user i is a complex Gaussian vector with covariance matrix $\mathbf{Q_i}$.

Our results make use of the following theorems. The proofs can be found in [8].

Theorem 1: Let $f(\mathbf{Q}) \triangleq \mathrm{E}_{\mathbf{H}} \log|\mathbf{A} + \mathbf{HQH}^{\dagger}|$ where $\mathbf{Q}$ is an $n \times n$ covariance matrix and $\mathrm{trace}(\mathbf{Q}) \leq P$. Further, let the elements of $\mathbf{H}$ be i.i.d. zero mean complex Gaussians. Then, for any non-negative definite $\mathbf{A}$, $f(\mathbf{Q})$ is maximized for $\mathbf{Q} = \frac{P}{n}\mathbf{I}$.

Theorem 2: For $f(\mathbf{Q})$ as defined in Theorem 1, assume the rows of $\mathbf{H}$ are correlated ($\mathbf{H}[i.] \sim \tilde{N}(\mathbf{0},\Sigma_{\mathbf{H}})$) but the columns are i.i.d. Then as in Theorem 1, for any non-negative definite $\mathbf{A}$, and any covariance matrix Σ, $f(\mathbf{Q})$ is maximized for $\mathbf{Q} = \frac{P}{n}\mathbf{I}$.

Theorem 3: For $f(\mathbf{Q})$ as defined in Theorem 1, assume the columns of $\mathbf{H}$ are correlated ($\mathbf{H}[.i] \sim \tilde{N}(\mathbf{0},\Sigma_{\mathbf{H}})$) but the rows are i.i.d. Then, for any non-negative definite $\mathbf{A}$, the $\mathbf{Q}$ that maximizes $f(\mathbf{Q})$ has the same eigenvectors as $\Sigma_{\mathbf{H}}$.

The channel models with i.i.d. columns and correlated rows or vice versa come straight from the 'one-ring' fade model of a cellular system as described in [9]. The i.i.d. columns and correlated rows describe a system where the Base Station (BS) receiver is unobstructed while the SU transmit antennas are surrounded by local scatterers located on a circle centered around the SU. The scatterers decorrelate the fades associated with any two distinct transmit antennas (distinct columns of $\mathbf{H_i}$) but the fades associated with different receive antennas at the BS (distinct rows of $\mathbf{H_i}$) are still correlated. Similarly, i.i.d. rows and correlated columns correspond to a system where the BS receiver is located close to the ground and surrounded by scatterers that decorrelate the fades associated with different receive antennas. However the physical size constraint of the mobile unit forces the separation between its transmit antennas to be less than the decorrelating distance.

Similar to the approach in [5], we characterize the boundary points of the capacity region by maximizing $\boldsymbol{\mu}.\mathbf{R}$ for $\mu_i \in (0,1], 1 \leq i \leq K$ and $\sum_{i=1}^{K} \mu_i = 1$. For each permutation function π_i let us define the set $\Pi_i \triangleq \{\boldsymbol{\mu} : \mu_{\pi_i(1)} \leq \mu_{\pi_i(2)} \cdots \leq \mu_{\pi_i(K)}\}$. Using the results in [5] for the constant channel we can show that if $\boldsymbol{\mu} \in \Pi_i$, then $\boldsymbol{\mu}.\mathbf{R}$ is maximized by successive decoding in the same order. That is, user $\pi_i(1)$ is decoded first and user $\pi_i(K)$ is decoded last.

Without loss of generality, let $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_K$. Then the corresponding boundary point of the capacity region is given by $(R_1, R_2, \cdots, R_K)$ where

$$R_i = \mathrm{E}[\log\left(\frac{|\mathbf{I} + \sum_{j=1}^{i} \mathbf{H_j}\mathbf{Q_j}\mathbf{H_j}^{\dagger}|}{|\mathbf{I} + \sum_{k=1}^{i-1} \mathbf{H_k}\mathbf{Q_k}\mathbf{H_k}^{\dagger}|}\right)]$$

and $\mathbf{Q_1}, \mathbf{Q_2}, \cdots, \mathbf{Q_K}$ are the optimal covariances that maximize $\boldsymbol{\mu}.\mathbf{R}$. We rewrite the maximization problem as

$$\max_{\mathbf{Q_i}:\mathrm{trace}(\mathbf{Q_i}) \leq P_i, 1 \leq i \leq K} \sum_{i=1}^{K-1} (\mu_i - \mu_{i+1}) \mathrm{E}[\log |\mathbf{I} + \sum_{j=1}^{i} \mathbf{H_j}\mathbf{Q_j}\mathbf{H_j}^{\dagger}|] + \mu_K \mathrm{E}[\log |\mathbf{I} + \sum_{j=1}^{K} \mathbf{H_j}\mathbf{Q_j}\mathbf{H_j}^{\dagger}|]. \tag{3}$$

Note that all the coefficients $(\mu_i - \mu_{i+1})$ and μ_K are positive. So if each term is maximized by the same value of Q_i the optimal Q_i is fixed irrespective of $\boldsymbol{\mu}$ and the decoding order.

We wish to characterize the shape of our capacity region as a function of the channel correlations. Recall that the Gaussian MAC capacity region with no fading is a polymatroid [10]. For any decoding order π_i, all $\boldsymbol{\mu} \in \Pi_i$ lead to the same rate point (a vertex) on the boundary of the capacity region. With fading and perfect CSIT and CSIR however the

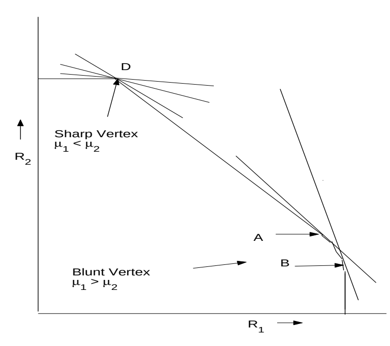

Fig. 1. Two User Capacity Region

boundary of the capacity region traced by $\boldsymbol{\mu} > \mathbf{0}$ is strictly convex. In

order to characterize the shape of the capacity region we use the following definition. We say that a decoding order π_i corresponds to a **sharp vertex** if all $\boldsymbol{\mu} \in \Pi_i$ lead to the same unique rate point on the boundary of the capacity region. A sharp vertex implies that the same transmit strategy is optimal for all $\boldsymbol{\mu} \in \Pi_i$. Otherwise we say it corresponds to a **blunt vertex**. Figure 1 shows a two user capacity region with a sharp vertex and a blunt vertex.

Next we list our main results. The proofs are based on the theorems stated earlier and are included in the full paper [8].

III. Results

A. General Correlated Channels

In general for correlated channels the optimal transmit strategy for a given decoding order Π_i is a function of the actual weights $\boldsymbol{\mu}$ and the capacity region has blunt vertices. Thus the boundary of the capacity region traced by $\boldsymbol{\mu} > \mathbf{0}$ is in general neither strictly convex nor a polymatroid. In fact we show that while it *can* be a polymatroid in several cases of practical interest, it can never be strictly convex. Characterizing the optimal transmit strategy for each $\boldsymbol{\mu} \in \Pi_i$ is in general an intractable problem. However, as we see next, for many cases of practical interest the optimal transmit strategy can be determined.

B. I.i.d. fades

If the fades from each of the i^{th} user's n_{T_i} transmit antennas to each of the n_R receive antennas are i.i.d. then the input covariance matrix for the i^{th} user that maximizes $\boldsymbol{\mu}.\mathbf{R}$ is given by $Q_i = \frac{P_i}{n}\mathbf{I}$, irrespective of the decoding order, $\boldsymbol{\mu}$, and the channel correlations of the rest of the users. Further if all users have i.i.d. fading channels, the capacity region is a polymatroid.

C. Correlated rows and i.i.d. columns

If the channel matrix for the i^{th} user has correlated rows and i.i.d. columns, then as for the i.i.d. fades, the input covariance matrix for the i^{th} user that maximizes $\boldsymbol{\mu}.\mathbf{R}$ is given by $Q_i = \frac{P_i}{n}\mathbf{I}$, irrespective of the decoding order, $\boldsymbol{\mu}$, and the channel correlations of the rest of the users. Also if all users have correlated rows and i.i.d. columns fading channels, the capacity region is a polymatroid.

D. *Correlated columns and i.i.d. rows*

If the channel matrix for the i^{th} user has i.i.d. rows and correlated columns, then the the input covariance matrix for the i^{th} user that maximizes $\boldsymbol{\mu}.\mathbf{R}$ has the same eigenvectors as that of the covariance matrix of the rows of the channel matrix, irrespective of the decoding order, $\boldsymbol{\mu}$, and the channel correlations of the rest of the users. However, the optimal eigenvalues of the input covariance matrix depend on the decoding order, $\boldsymbol{\mu}$, and the channel correlations of the rest of the users. The vertices are blunt and the capacity region may not be a polymatroid.

E. *Strong Modes*

For strongly correlated channels the disparity between the eigenvalues of the channel covariance matrix can be large. It was shown in [4] for a single user, that if the disparity between the two largest eigenvalues of the channel covariance matrix is large enough, the optimal input covariance matrix has unit rank (beamforming). For the multiple access channel, we show that if the i^{th} user's channel correlations satisfy the single user necessary condition for optimality of beamforming given in [4], then the input covariance matrix for the i^{th} user that maximizes $\boldsymbol{\mu}.\mathbf{R}$ has unit rank irrespective of the decoding order, $\boldsymbol{\mu}$, and the channel correlations of the rest of the users. Further if all users satisfy the single user necessary condition for optimality of beamforming the capacity region is a polymatroid.

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