Adaptive Multirate CDMA for Uplink Throughput Maximization

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Abstract

We determine the optimal adaptive rate and power control strategies to maximize the total throughput in a multirate CDMA system. The total throughput of the system provides a meaningful baseline in the form of an upperbound to the throughput achievable with additional restrictions imposed on the system to guarantee fairness. Peak power and instantaneous bit energy-to-noise spectral density constraints are assumed at the transmitter with matched filter detection at the receiver. Our results apply to frequency selective fading in so far as the bit energy-to-equivalent noise power spectral density ratio definition can be used as the Quality of Service metric. The bit energy-to-equivalent noise power spectral density ratio metric coincides with the BER metric under the assumption that the processing gains and the number of users are high enough that self-interference can be neglected. We first obtain results for the case where the rates available to each user are unrestricted, and we then consider the more practical scenario where each user has a finite discrete set of rates. An upper bound to the maximum average throughput is obtained and evaluated for Rayleigh fading. Sub-optimal low-complexity schemes are considered to illustrate the performance tradeoffs between optimality and complexity. We also show that the optimum rate and power adaptation scheme with unconstrained rates is in fact just a rate adaptation scheme with fixed transmit powers, and it performs significantly better than a scheme that uses power adaptation alone.

I. Introduction

Multirate DS-CDMA and adaptive modulation form the foundations for future wireless communication systems, and there is much previous work in this area [1] - [12]. For point to point, single user narrowband wireless communications, adapting the transmit rate and power to the channel fade variations is a well known strategy to enhance the average throughput [1]-[7]. Multirate DS-CDMA schemes offer a simple way to provide different data rates to different users in a system. However, adaptive CDMA remains a relatively unexplored area of research. "Adaptation" in the context of CDMA systems has been mostly synonymous with power control. Wireless networks of the past have been designed primarily for voice traffic. Due to the delay intolerant nature of voice, rate adaptation is not desirable. Thus, in the past, the goal for system design has been to provide constant rate communications, using power adaptation to compensate for channel fades and propagation path loss. However, wireless data is viewed as a vast source of revenue for future wireless systems. The delay tolerant nature of data traffic allows rate adaptation. This leads to a significant shift in the objectives governing the design of wireless systems. Instead of traditional power adaptation schemes that maintain a constant rate, the focus is now on joint rate and power adaptation schemes that maximize the total throughput. CDMA schemes lend themselves to rate adaptation in a simple manner by using multiple codes, multiple processing gains, or multirate modulations. The throughput gains with multirate CDMA schemes have been studied in [13][14][15][16][17]. Wasserman and Oh [13] considered optimal (throughput maximizing) dynamic spreading gain control with perfect power control. They also considered optimal joint rate and power adaptation subject to a peak transmit power constraint and a maximum interference constraint [14]. Adaptive code rates with multiple orthogonal codes were considered in [15]. Hashem and Sousa [16] showed that limiting the increase in power to compensate for multipath fading, and getting the extra gain required by reducing the transmission rate, can increase the total throughput by about 231% for flat Rayleigh fading. Kim and Lee [17] showed the power gains achieved by the same scheme, and also considered truncated rate adaptation.

This paper is motivated by the need to estimate the *maximum* throughput that can be achieved through joint rate and power adaptation with perfect channel information, subject to an instantaneous minimum bit energy-to-equivalent noise power spectral density ratio threshold and with conventional single user matched filter detection for each user at the base station. In practice any system will have additional constraints arising out of the need to be "fair" to users in deep fades, other concerns like prolonging battery life (average power constraint), or additional QOS requirements (delay constraints). For example, Wasserman and Oh [14] consider the maximum throughput subject to an additional constraint in the form of a limit on the intercell interference. Throughput for a practical system will also depend on the reliability and delay of the feedback channel, rate of adaptation (compared to the rate of change of fade levels), and errors in the channel estimates. All these factors will reduce the throughput of an adaptive multirate CDMA scheme in practice. However, we evaluate the maximum throughput without these additional constraints, to serve as an upper bound on the performance of adaptive multirate CDMA schemes in practice. We restrict our attention to multiple processing gain schemes since it has been shown that multirate modulation schemes have significantly worse performance for high data rate users. Multiple code schemes where the set of codes available to each user are orthogonal within themselves but not across users, have almost the same performance as multiple processing gain schemes [19]. However, multiple code schemes suffer from the disadvantage of a high peak-to-mean envelope power ratio. The orthogonality of codes is also compromised in frequency selective fading [19]. Other multirate schemes like Parallel Combinatory Spread Spectrum [10] and multiple chip-rate [11] have been proposed but aren't considered as viable. The conventional matched filter receiver, although suboptimal for multiuser detection, remains popular because of its simplicity. We use a maximum power constraint since any systems in practice will necessarily have an upper limit on the transmit power, especially on the reverse link. The peak transmit power is also limited by the tradeoff between the power amplifier efficiency and the desired peak-to-average ratio. Our QOS measure is the minimum bit energy-to-equivalent noise spectral density ratio, E_b/N_e.

The remainder of this paper is organized as follows. The system model is described in the next section. A general problem definition is given in Section III. The same section also introduces the set of constraints that are used in the later sections. Section IV presents the optimal rate and power adaptation scheme assuming unlimited, continuously-variable rates. The effect of limiting the maximum achievable instantaneous rates to a finite value is considered in Section V. Section VI is devoted to the derivation of an analytical upper bound to estimate the maximum *average* throughput for the system with unrestricted rates. Section VII further limits the rates to a set of discrete values, and presents the optimum rate and power adaptation scheme under this added constraint. The maximum average throughputs achievable with optimum rate or optimum power adaptation alone are derived in Section VIII. Comparisons between the schemes and numerical results are presented in Section IX. Section X concludes this paper.

II. System Model

Our system model is similar to the one used in [17]. We consider a variable bit duration CDMA system with K users. The system uses BPSK with coherent demodulation. A minimum bit energy-to-noise spectral density threshold (as defined later in (1)) must be met for a user to transmit on the channel. We assume that the channel is frequency-selective with respect to the spreading bandwidth, and is affected by slow fading (assumed constant over a bit time), additive white Gaussian noise (AWGN), and multiple access interference (MAI) due to other users. For simplicity we look at a single cell system. However the model can be extended to multiple cell systems by incorporating an out-of-cell interference coefficient [21]. The user's channel access is assumed to be asynchronous.

The bit energy-to-equivalent noise spectral density ratio $\frac{E_b}{N_e}$ at the L-finger RAKE receiver output for user i is given ([17] [19]) as

$$\frac{E_b}{N_e} = \frac{\frac{P_i(\overline{\chi},\overline{r})}{n_i(\overline{\chi},\overline{r})}}{\frac{2}{3}\sum_{k\in I-\{i\}} P_k(\overline{\chi},\overline{r}) + \frac{N_o}{T_c}}, \tag{1}$$

with the symbols defined as follows:

- $I = \{1, 2, \cdots, K\}$ is the index set of users.
- $\overline{r} = \{r_1, r_2, \cdots, r_K\}$ is the vector of distances of the K users from the base station.
- N_o is the one-sided power spectral density of AWGN.
- T_c is the chip duration.
- $\overline{\chi} = \{\chi_1, \chi_2, \cdots, \chi_K\}$ is the vector of the users' equivalent channel power fade levels at the output of the L-finger RAKE receiver, defined as

$$\chi_i = \sum_{l=1}^{L} \chi_{i,l}, \tag{2}$$

where $\chi_{i,l}$ is the channel power gain due to multipath fading for user i on the l^{th} path.

- $P_i(\overline{\chi},\overline{r})$, the received power of the i^{th} user at the base station, given by

$$P_i(\overline{\chi},\overline{r}) = S_i(\overline{\chi},\overline{r}) g_i(r_i) \chi_i \tag{3}$$

where $g_i(r_i)$ is the propagation path loss, and $S_i(\overline{\chi},\overline{r})$ is the total transmitted power of the i^{th} user.

- $n_i(\overline{\chi},\overline{r})$ is the rate of the i^{th} user, defined as

$$n_i(\overline{\chi},\overline{r}) = \frac{T_c}{T_i^b(\overline{\chi},\overline{r})} \tag{4}$$

where $T_i^b(\overline{\chi},\overline{r})$ is the bit duration for user i.

We use the bit energy-to-equivalent noise spectral density ratio (1) as our measure of QOS. For flat fading channels the probability of error can be obtained as $Q(\sqrt{2E_b/N_e})$. For frequency selective channels, when the processing gain is sufficiently high that self interference from various multipath components can be ignored, then as in the flat fading case, the E_b/N_e expression (1) leads to a good approximation to the probability of error.

From equation (1), the data rate for user i can be expressed as

$$n_i(\overline{\chi},\overline{r}) \leq \frac{3}{2(E_b/N_e)_o}\frac{P_i(\overline{\chi},\overline{r})}{\sum_{k\in I-\{i\}} P_k(\overline{\chi},\overline{r}) + \frac{3N_o}{2T_c}}, \tag{5}$$

where $(E_b/N_e)_o$ is the target bit energy-to-equivalent noise spectral density ratio.

III. Problem Definition and Constraints

Our goal is to maximize the total throughput of the system averaged over the fading distributions of the users, subject to a peak transmit power constraint and a target $(E_b/N_e)_o$ constraint. The total throughput is defined as the sum of the data rates of all users. The target E_b/N_e constraint implies that a user can transmit on the channel only if his instantaneous E_b/N_e is above the specified target level $(E_b/N_e)_o$. We assume the same target $(E_b/N_e)_o$ for all users. The peak transmit power is usually determined by the transmitter hardware. However, depending upon the propagation path loss and the channel fades, different users will have different peak received power constraints

$$P_{i,max}(\chi_i, r_i) = g_i(r_i)\chi_i S_{i,max}.$$

Subject to these constraints, we wish to find the optimal rate and power adaptation on the fading channel that maximizes the average total throughput. Note that since our power and $(E_b/N_e)_o$ constraints are instantaneous (rather than average), average throughput maximization is the same as instantaneous throughput maximization for each fade vector. The general optimization problem is therefore as follows:

Find the optimal rate and power adaptation to maximize the instantaneous throughput

$$T_{opt}(\overline{\chi},\overline{r}|c_1,c_2\dots c_n) = \max_{c_1,c_2\dots c_n}\sum_{k\in I} n_k(\overline{\chi},\overline{r})$$

subject to the constraints $c_1, c_2, \dots, c_n$.

The constraints we shall use in different sections are provided here for reference.

$c_1:$	$0 \leq P_i(\overline{\chi},\overline{r}) \leq P_{i,max}(\chi_i,r_i) = g_i(r_i)\chi_i S_{i,max}$	$\forall i \in I$	(peak power)
$c_2:$	when $P_i(\overline{\chi},\overline{r}) > 0$ then $\frac{\frac{P_i(\overline{\chi},\overline{r})}{n_i(\overline{\chi},\overline{r})}}{\sum_{k\in I-\{i\}} P_k(\overline{\chi},\overline{r})+\frac{3N_o}{2T_c}} \geq \frac{2}{3}(E_b/N_e)_o$	$\forall i \in I$	(min.E_b/N_e)
$c_3:$	$0 \leq n_i(\overline{\chi},\overline{r}) \leq \infty$	$\forall i \in I$	(unlimited rates)
$c_3':$	$0 \leq n_i(\overline{\chi},\overline{r}) \leq M \leq 1$	$\forall i \in I$	(limited rates)
$c_4:$	$n_i(\overline{\chi},\overline{r}) \in R^+$	$\forall i \in I$	(continuous rates)
$c_4':$	$n_i(\overline{\chi},\overline{r}) \in \{1,\frac{1}{2},\frac{1}{3},\cdots\}\cup\{0\}$	$\forall i \in I$	(discrete rates)

Note that while c_3 and c_4 look similar, they represent different constraints. c_3 and c_3' distinguish the case where the rates are bounded from the case where the rates can be arbitrarily large. On the other hand, we use c_4 and c_4' to distinguish the case where the rates can take values over a continuous set from the case where the rates can only take values over a discrete set. In practical multiple processing gain CDMA systems, a user's bit duration $T_i^b(\overline{\chi},\overline{r})$ must be an integer multiple of the chip duration T_c to prevent bandwidth expansion. Some processing gain ($\frac{1}{M}$) may also be desirable to suppress self-interference from multipath components. Note that the E_b/N_e expression in (1) assumes that the processing gain is high enough that self-interference can be neglected. Thus, a users' rate $n_i(\overline{\chi},\overline{r}) = \frac{T_c}{T_i(\overline{\chi},\overline{r})}$ can only take discrete values given by:

$$n_i(\overline{\chi},\overline{r}) \in \{1, \frac{1}{2}, \frac{1}{3}, \cdots\} \tag{6}$$

and is bounded by:

$$n_i(\overline{\chi},\overline{r}) \leq M \leq 1. \tag{7}$$

where $\frac{1}{M}$ is the minimum processing gain allowed for the system.

Thus, practical multiple processing gain CDMA systems are represented by constraints c_3' and c_4'. However, for simplicity, we first consider a system with constraints c_1, c_2, c_3, c_4, where the users' transmit rates are not restricted to a discrete or bounded set. This gives an upper bound on the performance of systems restricted to a finite, discrete set of rates. For these restricted systems, we use constraints c_1, c_2, c_3', c_4'. Note that we expect the upper bounds to be meaningful because even with unconstrained rates (c_3, c_4), for parameter values of practical interests the probability of $n_i(\overline{\chi},\overline{r}) > 1$ is small [17]. This is also verified by numerical results presented in Section IX.

IV. Optimal Unlimited Continuous Rate and Power Adaptation Scheme

In this section we treat the users' rates $n_i(\overline{\chi},\overline{r})$ as continuous variables that take values over the entire range of positive real numbers. The resulting maximum throughput gives an upper bound on the performance of practical systems. Under this assumption we present the following propositions:

Proposition 1: The optimal solution that maximizes the average total throughput is such that

$$P_k(\overline{\chi},\overline{r}) \in \{0, P_{k,max}(\chi_k, r_k)\} \qquad \forall k \in I.$$

That is, either a user does not transmit, or he transmits at full power.

Proof: From the $(E_b/N_e)_o$ constraint (c_2) we get

$$n_i(\overline{\chi},\overline{r}) = \frac{3}{2(E_b/N_e)_o} \frac{P_i(\overline{\chi},\overline{r})}{\sum_{k\in I-\{i\}} P_k(\overline{\chi},\overline{r}) + \frac{3N_o}{2T_c}}. \tag{8}$$

Note that we replaced the inequality in (c_2) by an equality. This is because in a CDMA system using conventional matched filter receivers for each user, MAI is minimized when users transmit just enough power to meet their QOS constraint. Note that according to our problem statement there is no benefit in reducing (E_b/N_e) below the target value $(E_b/N_e)_o$. The total instantaneous throughput can therefore be expressed as

$$T(\overline{\chi},\overline{r}) = \sum_{i\in I} n_i(\overline{\chi},\overline{r}) = \frac{3}{2(E_b/N_e)_o}\sum_{i\in I}\frac{P_i(\overline{\chi},\overline{r})}{\sum_{k\in I-\{i\}} P_k(\overline{\chi},\overline{r}) + \frac{3N_o}{2T_c}}. \tag{9}$$

Differentiating twice with respect to $P_i(\overline{\chi},\overline{r})$ we obtain

$$\frac{\partial^2 T(\overline{\chi},\overline{r})}{\partial P_i(\overline{\chi},\overline{r})^2} = \frac{3}{(E_b/N_e)_o}\sum_{j\in I-\{i\}}\frac{P_j(\overline{\chi},\overline{r})}{\left(\sum_{k\in I-\{j\}} P_k(\overline{\chi},\overline{r}) + \frac{3N_o}{2T_c}\right)^3}. \tag{10}$$

But this is always non-negative. Hence $T(\overline{\chi},\overline{r})$ is a convex function of $P_i(\overline{\chi},\overline{r})$ and the maximum value will always lie at the boundary. This completes the proof. ■

Proposition 1 points to a tradeoff between a user's transmit power, his throughput, the interference he adds to the system, and the total system throughput. By increasing his power a user increases his own throughput while everyone else's throughput gets reduced due to additional MAI contributed by him. On the other hand, by reducing his power a user decreases his own throughput as well as the MAI, which allows other users to increase their throughputs. The convexity of the *total* throughput as a function of a user's transmit power proved in Proposition 1 can be interpreted as follows: As a user increases his power starting from zero, the total throughput may initially get reduced as the loss of throughput due to added MAI dominates the increase in throughput due to the new user. However, as the user keeps increasing his power, eventually the increase in throughput due to him more than compensates for the loss of throughput due to added MAI for others. Beyond this point, a higher power for the new user only increases the total system throughput.

Next we define an ordering on users that will be helpful in characterizing the throughput maximizing solution.

Best Users (Definition) : Without loss of generality we assume that

$$P_{i,max}(\chi_i, r_i) > P_{j,max}(\chi_j, r_j) \qquad \forall i \le j \quad i,j \in I. \tag{11}$$

Under this assumption we define the *best* k users as the *first* k users in the index set I. Note that these are the *best* users since they have the greatest *received* powers at the base station when transmitting at their maximum power. Also note that we used strict inequality in (11). We can do this because the channel fades and the propagation loss are continuous variables and therefore with probability 1, the peak received powers of any two users are unequal.

In light of Proposition 1, we can write the optimum instantaneous throughput as

$$T_{opt}(\overline{\chi},\overline{r}|c_1,c_2,c_3,c_4) = D\sum_{i\in I_{opt}}\frac{P_{i,max}(\chi_i,r_i)}{\sum_{k\in I_{opt}-\{i\}} P_{k,max}(\chi_k,r_k) + C}\ , \tag{12}$$

where, for notational convenience, we define $D = \frac{3}{2(E_b/N_e)_o}$ and $C = \frac{3N_o}{2T_c}$. $I_{opt} \subset I$ is the set of users transmitting at their peak powers for maximum throughput. We still need to find I_{opt} for the optimum solution. There are $2^K - 1$ non-empty subsets of I, and the throughput for each can be found according to equation (12) with complexity $\sim O(K)$. So all the possibilities for I_{opt} can be tested with complexity $\sim O(K2^K)$. However, as we prove shortly, we need to consider only *K-1* possibilities. This leads to the next proposition.

Proposition 2: $\exists \;\; k_{opt}, 0 < k_{opt} \leq K$, such that the optimum solution can be achieved with $I_{opt} = \{1, 2, \dots, k_{opt}\}$. That is, the optimum solution is achieved for the k_{opt} *best* users, with received powers $P_{i,max}(\chi_i, r_i), \;\; 1 \leq i \leq k_{opt}$.

Proof: A proof by contradiction is presented as follows:

Let k_{opt} be the *minimum* number of users that need to transmit simultaneously to achieve the maximum possible throughput. Further, let user i be one of these k_{opt} users, such that $i > k_{opt}$, i.e. user i is not one of the k_{opt} *best* users. Now, user i's received power at the base station can be achieved by any of the k_{opt} *best* users (from the definition of the *best* users). Hence it is possible to replace user i with any of the k_{opt} *best* users that are not already included in the optimum solution as follows. Let the better user transmit only enough power to achieve the same received power as user i. This ensures that the other users' throughputs are not affected. Moreover the better user also gets the same throughput as user i in the optimal solution. However, note that in order to achieve the same received power as user i, the better user who replaces him, must transmit at less than his peak power. Now let us use the result of Proposition 1. The total throughput is a convex function of the better user's power. So it must be possible to increase the total throughput by either increasing or decreasing his power. Since he is not operating at his peak power, he can transmit at a higher or lower power. Thus, the solution we started with cannot be optimal. The contradiction completes the proof. ■

In the light of Propositions 1 and 2, the optimum unlimited continuous rate and power adaptation scheme is as follows: Given a channel power fade level vector $\overline{\chi}$ and a user distance vector $\overline{r}$ compute the peak received powers $P_{i,max}(\chi_i, r_i)$ for each user (as given in (c_2)) and sort them according to (11). Next, find the throughputs achieved by the n *best* users transmitting at their peak transmit powers as

$$T_n(\overline{\chi}, \overline{r}|c_1, c_2, c_3, c_4) = D \sum_{i=1}^{n} \frac{P_{i,max}(\chi_i, r_i)}{\sum_{k \in I_{opt}-\{i\}} P_{k,max}(\chi_k, r_k) + C}$$

for $1 \leq n \leq K$. Then

$$T_{opt}(\overline{\chi}, \overline{r}|c_1, c_2, c_3, c_4) = \max_{1 \leq n \leq K} T_n(\overline{\chi}, \overline{r}|c_1, c_2, c_3, c_4)$$

and

$$k_{opt} = \arg \max_{1 \le n \le K} T_n(\overline{\chi}, \overline{r} | c_1, c_2, c_3, c_4).$$

$T_n(\overline{\chi}, \overline{r} | c_1, c_2, c_3, c_4)$ can be evaluated with complexity $\sim O(K)$ for each n. Thus the optimum rates and powers are found with a computational complexity $\sim O(K^2) \ll O(K2^K)$ for large K. We use the optimum rate and power adaptation strategy to evaluate the maximum average throughput for our model in Section IX.

It is interesting to draw a comparison between the optimal solution characterized by the results of Propositions 1 and 2 and the optimal solution obtained by Wasserman and Oh in [14]. In particular, note that although [14] used an additional constraint on the intercell interference and did not assume target E_b/N_e requirements, the results of Propositions 1 and 2 were found to hold in [14] as well.

V. Optimum Limited Continuous Rate and Power Adaptation Scheme

In the previous section we allowed an unlimited rate for every user. However, as mentioned earlier, to prevent bandwidth expansion, a user's bit duration cannot be smaller than the chip duration. The maximum rate may also be limited by the minimum processing gain allowed by the system. It is therefore interesting to see how throughput is affected when we limit the rates available to a user. In this section we wish to estimate the loss of throughput due to limiting the maximum rate available to each user. This loss is defined as

$$\triangle T = T_{opt}(\overline{\chi}, \overline{r} | c_1, c_2, c_3, c_4) - T_{opt}(\overline{\chi}, \overline{r} | c_1, c_2, c_3', c_4).$$

Note that we still assume that the user's rate is a continuous variable (c_4).

We wish to find $T_{opt}(\overline{\chi}, \overline{r} | c_1, c_2, c_3', c_4)$. Let the optimum throughput be achieved with a minimum of k_{trans} users transmitting. Without loss of generality, these can be assumed to be the k_{trans} *best* users (Proposition 2). Under the assumption in (11) the index set of these k_{trans} users is $I_{trans} = \{1, 2, \cdots, k_{trans}\}$. Let us divide the set I_{trans} into three subsets I_M, I_P and I_R, such that $I_M \cup I_P \cup I_R = I_{trans}$ and $I_M \cap I_P = I_M \cap I_R = I_P \cap I_R = \phi$. These subsets are defined as follows:

$$\forall i \in I_M, \qquad n_i(\overline{\chi}, \overline{r}) = M, \tag{13}$$

$$\forall i \in I_P, \qquad P_i(\overline{\chi}, \overline{r}) = P_{i,max}(\chi_i, r_i) \quad \& \quad n_i(\overline{\chi}, \overline{r}) < M, \tag{14}$$

$$I_R = I_{trans} - (I_M \cup I_P). \tag{15}$$

That is, I_M is the index set of users operating at the rate boundary (transmitting at rate M), I_P is the index set of users at the power boundary (transmitting at peak power $S_{i,max}$) that are not at the rate boundary, and I_R is the index set of the remaining users that need to transmit to achieve the maximum throughput. Let the number of users in each set be k_M, k_P, and k_R respectively. The following propositions characterize the optimal powers and rates of users in these sets.

Proposition 3: The optimal solution is such that $\forall i, j \in I_{trans}$, if $P_i(\overline{\chi}, \overline{r}) > P_j(\overline{\chi}, \overline{r}) > 0$, then $i \in I_M \cup I_P$.

Proof: Let there be users i and j such that $P_i(\overline{\chi}, \overline{r}) > P_j(\overline{\chi}, \overline{r}) > 0$, and $i \notin I_M \cup I_P$. Let the sum of the optimum received powers of all other users be

$$\sum_{k \in I - \{i,j\}} P_i(\overline{\chi}, \overline{r}) = P_{rest}. \tag{16}$$

Also let the sum of the optimum received powers of users i and j be

$$P_i(\overline{\chi}, \overline{r}) + P_j(\overline{\chi}, \overline{r}) = P_{ij}. \tag{17}$$

Now the MAI seen by users other than i and j depends only on P_{ij} and not on $P_i(\overline{\chi}, \overline{r})$ or $P_j(\overline{\chi}, \overline{r})$ individually. Therefore, for a fixed P_{ij}, if we change $P_i(\overline{\chi}, \overline{r})$ then other users, and their corresponding throughputs, are unaffected. So let us maximize the total throughput of users i and j subject to (17). This throughput is given as

$$\begin{aligned} T_{ij}(\overline{\chi}, \overline{r}) &= \frac{DP_i(\overline{\chi}, \overline{r})}{P_{rest} + P_j(\overline{\chi}, \overline{r}) + C} + \frac{DP_j(\overline{\chi}, \overline{r})}{P_{rest} + P_i(\overline{\chi}, \overline{r}) + C} && (18) \\ &= \frac{DP_i(\overline{\chi}, \overline{r})}{P_{rest} + P_{ij} - P_i(\overline{\chi}, \overline{r}) + C} + \frac{D(P_{ij} - P_i(\overline{\chi}, \overline{r}))}{P_{rest} + P_i(\overline{\chi}, \overline{r}) + C}. && (19) \end{aligned}$$

Differentiating with respect to $P_i(\overline{\chi}, \overline{r})$ we obtain

$$\begin{aligned} \frac{\partial T_{ij}(\overline{\chi}, \overline{r})}{P_i(\overline{\chi}, \overline{r})} &= D(P_{rest} + P_{ij} + C)\left(\frac{1}{(P_{rest} + P_{ij} - P_i(\overline{\chi}, \overline{r}) + C)^2} - \frac{1}{(P_{rest} + P_i(\overline{\chi}, \overline{r}) + C)^2}\right) \\ &= D(P_{rest} + P_{ij} + C)\left(\frac{1}{(P_{rest} + P_j(\overline{\chi}, \overline{r}) + C)^2} - \frac{1}{(P_{rest} + P_i(\overline{\chi}, \overline{r}) + C)^2}\right) \\ &> 0 && (20) \end{aligned}$$

But for the optimal solution the derivative must be zero. So the proof follows by contradiction. ■

Hence we can increase the total throughput by reducing the weaker user's power and increasing the stronger user's power such that the sum of their received powers is the same. We can keep doing this until the stronger user hits the rate or power boundary, or the weaker user's power goes to zero. Note that even though the derivative found above is zero for $P_i(\overline{\chi}, \overline{r}) = P_j(\overline{\chi}, \overline{r})$, this point only represents a minima, as can be easily verified from the second derivative.

Proposition 4: The optimum solution is such that

$$\forall i,j \in I_M \quad P_i(\overline{\chi},\overline{r}) = P_j(\overline{\chi},\overline{r}) = P_M(\overline{\chi},\overline{r}), \tag{21}$$

$$\forall i \in I_P \quad P_i(\overline{\chi},\overline{r}) = P_{i,max}(\chi_i, r_i), \tag{22}$$

$$k_R \leq 1, \tag{23}$$

$$\forall i \in I_P \quad P_M(\overline{\chi},\overline{r}) \geq P_i(\overline{\chi},\overline{r}) \geq P_R, \tag{24}$$

where P_R is the received power of the user in I_R (i.e. not at his rate or power boundary). By (23) there can be at most one such user. If there is no such user we define $P_R = 0$.

Proof: First, consider two users on the rate boundary $i, j \in I_M$. This implies that $n_i(\overline{\chi},\overline{r}) = n_j(\overline{\chi},\overline{r})$. Substituting from (8) we get that

$$\frac{DP_i(\overline{\chi},\overline{r})}{\sum_{k\in I-\{i,j\}} P_k(\overline{\chi},\overline{r}) + P_j(\overline{\chi},\overline{r}) + C} = \frac{DP_j(\overline{\chi},\overline{r})}{\sum_{k\in I-\{i,j\}} P_k(\overline{\chi},\overline{r}) + P_i(\overline{\chi},\overline{r}) + C} \tag{25}$$

which proves (21). (22) follows from the definition of I_P. (23) follows from Proposition 3 and the fact that equal received powers correspond to a minimum in the total throughput as mentioned earlier. Since higher rates require higher powers, (24) is trivial. ■

Propositions 3 and 4 tell us that the maximum throughput with finite rates is achieved with k_M *best* users at the rate boundary, each with the same received power $P_M(\overline{\chi},\overline{r})$, the next k_P *best* users at the power boundary (transmitting at $S_{i,max}$), and at most one user with received power P_R that is not at his power or rate boundary. Thus we have $I_M = \{1, 2, \cdots, k_M\}$, $I_P = \{k_M + 1, k_M + 2, \cdots, k_M + k_P\}$, and $I_R = \{k_M + k_P + 1\}$. Assuming we know k_M and k_P we now want to find the optimum values of $P_M(\overline{\chi},\overline{r})$ and $P_R(\overline{\chi},\overline{r})$. From the peak power constraint we have

$$P_M(\overline{\chi},\overline{r}) \leq P_{k_M,max}(\chi_{k_M}, r_{k_M}), \tag{26}$$

and

$$P_R(\overline{\chi},\overline{r}) \leq P_{k_{trans},max}(\chi_{k_{trans}}, r_{k_{trans}}). \tag{27}$$

Define

$$P_p = \sum_{i\in I_p} P_i(\overline{\chi},\overline{r}) = \sum_{i\in I_p} P_{i,max}(\chi_i, r_i). \tag{28}$$

Then, from equation (8), for a user on the rate boundary we have

$$M = \frac{DP_M(\overline{\chi},\overline{r})}{P_p + (k_M - 1)P_M(\overline{\chi},\overline{r}) + P_R(\overline{\chi},\overline{r}) + C}, \tag{29}$$

which gives us

$$P_M(\overline{\chi},\overline{r})\left(\frac{D}{M}+1-k_M\right)=P_p+P_R(\overline{\chi},\overline{r})+C. \tag{30}$$

Now the total throughput can be written as

$$\begin{aligned}&T_{k_M,k_P}(\overline{\chi},\overline{r}|c_1,c_2,c_3',c_4)=k_M M\\&+\tfrac{DP_R(\overline{\chi},\overline{r})}{P_p+k_M P_M(\overline{\chi},\overline{r})+C}+\textstyle\sum_{i\in I_P}\tfrac{DP_{i,max}(\chi_i,r_i)}{k_M P_M(\overline{\chi},\overline{r})+P_p-P_{i,max}(\chi_i,r_i)+P_R(\overline{\chi},\overline{r})+C}.\end{aligned} \tag{31}$$

Substituting for $P_R(\overline{\chi},\overline{r})$ from (30) we get

$$\begin{aligned}&T_{k_M,k_P}(\overline{\chi},\overline{r}|c_1,c_2,c_3',c_4)=k_M M\\&+\tfrac{D\left(P_M(\overline{\chi},\overline{r})(\frac{D}{M}+1-k_M)-P_p-C\right)}{P_p+k_M P_M(\overline{\chi},\overline{r})+C}+\textstyle\sum_{i\in I_P}\tfrac{DP_{i,max}(\chi_i,r_i)}{P_M(\overline{\chi},\overline{r})(1+\frac{D}{M})-P_{i,max}(\chi_i,r_i)}.\end{aligned} \tag{32}$$

Differentiating the total throughput with respect to $P_M(\overline{\chi},\overline{r})$ gives us

$$\frac{1}{D}\frac{\partial T_{k_M,k_P}(\overline{\chi},\overline{r}|c_1,c_2,c_3',c_4)}{\partial P_M(\overline{\chi},\overline{r})}=\frac{(P_p+C)(\frac{D}{M}+1)}{(P_p+k_M P_M(\overline{\chi},\overline{r})+C)^2}-\sum_{i\in I_P}\frac{P_{i,max}(\chi_i,r_i)(1+\frac{D}{M})}{\left(P_M(\overline{\chi},\overline{r})(1+\frac{D}{M})-P_{i,max}(\chi_i,r_i)\right)^2}.$$

Rearranging terms and equating to zero we obtain

$$h(P_M(\overline{\chi},\overline{r}))\triangleq\sum_{i\in I_P}P_{i,max}(\chi_i,r_i)f(i,P_M(\overline{\chi},\overline{r}))=C+P_p, \tag{33}$$

where

$$f(i,P_M(\overline{\chi},\overline{r}))=\left(\frac{k_M+\frac{P_p+C}{P_M(\overline{\chi},\overline{r})}}{1+\frac{D}{M}-\frac{P_{i,max}(\chi_i,r_i)}{P_M(\overline{\chi},\overline{r})}}\right)^2. \tag{34}$$

Note that $f(i,P_M(\overline{\chi},\overline{r}))$ and therefore $h(P_M(\overline{\chi},\overline{r}))$ are monotonically decreasing functions of $P_M(\overline{\chi},\overline{r})$. So there exists only one solution to (33) that can easily be found numerically. Further, note that $P_M(\overline{\chi},\overline{r})$ is restricted as follows:

$$\begin{aligned}P_M(\overline{\chi},\overline{r}) &\leq P_{k_M,max}(\chi_{k_M},r_{k_M}), &(35)\\ P_M(\overline{\chi},\overline{r})\left(\frac{D}{M}+1-k_M\right) &\leq P_p+P_{k_{trans},max}(\chi_{k_{trans}},r_{k_{trans}})+C, &(36)\\ P_M(\overline{\chi},\overline{r})\left(\frac{D}{M}+1-k_M\right) &\geq P_p+C, &(37)\\ \text{if}\quad k_P\neq 0\quad\text{then}\quad P_M(\overline{\chi},\overline{r}) &> P_{k_M+1,max}(\chi_{k_M+1},r_{k_M+1}), &(38)\end{aligned}$$

where (35) follows from (26), (36) and (37) follow from (30) and (27), and (38) follows from the fact that a user on the power boundary cannot have a higher received power than a user on the rate boundary. Let the maximum and minimum possible values of $P_M(\overline{\chi},\overline{r})$ given by these constraints be

P_{min} and P_{max}. Define $\mathcal{T}(P)$ as the throughput achieved with $P_M(\overline{\chi}, \overline{r}) = P$. The optimal value of $P_M(\overline{\chi}, \overline{r})$ is therefore found as

$$\begin{aligned} P_M(\overline{\chi}, \overline{r}) &= P \text{ such that} \\ \mathcal{T}(P) &= \begin{cases} \max\{\mathcal{T}(P_{min}), \mathcal{T}(P_{max}), \mathcal{T}(h^{-1}(C+P_p))\} & \text{if } h(P_{min}) > C + P_p > h(P_{max}) \\ \max\{\mathcal{T}(P_{min}), \mathcal{T}(P_{max})\} & \text{otherwise.} \end{cases} \end{aligned} \quad (39)$$

Note that $h^{-1}()$ needs to be evaluated numerically. However, we need to evaluate it only when $h(P_{min}) > C + P_p > h(P_{max})$.

Thus the optimal rate and power adaptation scheme with finite continuous rates is as follows: Given a channel power fade level vector $\overline{\chi}$ and a user distance vector $\overline{r}$ compute the peak received powers $P_{i,max}$ for each user as $P_{i,max} = S_{i,max}\chi_i g_i(r_i)$ and sort them according to (11). Then the optimum throughput is found as

$$T_{opt}(\overline{\chi}, \overline{r}|c_1, c_2, c_3', c_4) = \max_{k_M \le \min(\frac{D}{M}+1, K), k_P \le K - k_M - 1} T_{k_M, k_P}(\overline{\chi}, \overline{r}|c_1, c_2, c_3', c_4). \quad (40)$$

We use this scheme to find the optimum average throughput for our system in Section IX.

VI. Analytical Upper bound for Unlimited Continuous Rate and Power Adaptation

The optimal scheme considered earlier gives us a way to find out the maximum *instantaneous* throughput achievable for a given channel fade vector and a given user location vector. However, an analytical expression for the maximum *average* (averaged out over the probability distribution of the channel fade vector) throughput is difficult to achieve. We can upper bound this optimal average throughput as follows:

$$T_{opt}(\overline{\chi}, \overline{r}|c_1, c_2) \le T_{opt}(\overline{\chi}, \overline{r}|c_1', c_2)$$

where

$${c_1}' : 0 \le P_i(\overline{\chi}, \overline{r}) \le P_{max}(\overline{\chi}, \overline{r}) = \max_{j \in I} P_{j,max}(\chi_j, r_j) = \max_{j \in I} S_{j,max}\chi_j g_j(r_j).$$

In other words, we bound the total throughput of the system by the throughput of a hypothetical system (call it the upperbound system) where all users are as good as the best/strongest user in the actual system. However, note that this is just a bound to the *actual* throughput of the *actual* system. In the actual system, the users are not equally strong.

The optimum rate and power allocation strategy gives us:

$$\begin{aligned} T_{opt}(\overline{\chi}, \overline{r}|c_1', c_2) &= \max_{0<k_{trans}\le K} \frac{Dk_{trans}P_{max}(\overline{\chi}, \overline{r})}{(k_{trans}-1)P_{max}(\overline{\chi}, \overline{r}) + C} \\ &= \begin{cases} \frac{DKP_{max}(\overline{\chi}, \overline{r})}{(K-1)P_{max}(\overline{\chi}, \overline{r})+C} & \text{if } P_{max}(\overline{\chi}, \overline{r}) < C \\ \frac{DP_{max}(\overline{\chi}, \overline{r})}{C} & \text{otherwise.} \end{cases} \end{aligned} \quad (41)$$

Note that if the peak received power is more than C then according to (41) only one user in the upperbound system should transmit. However, note that the *actual* system can also achieve this throughput since the strongest user in the actual system is as good as any user in the upperbound system. Thus the upper bound is tight in this case. To average out over $P_{max}(\overline{\chi}, \overline{r})$ we need the distribution of $P_{max}(\overline{\chi}, \overline{r})$.The cumulative distribution function of $P_{max}(\overline{\chi}, \overline{r})$

$$\begin{aligned} F_{P_{max}}(x) &= Prob(P_{max}(\overline{\chi}, \overline{r}) \le x) \\ &= Prob(S_{1,max}g_1(r_1)\chi_1 \le x \wedge S_{2,max}g_2(r_2)\chi_2 \le x \wedge \cdots \wedge S_{K,max}g_K(r_K)\chi_K \le x) \\ &= \prod_{i\in I} F_{\chi_i}(\frac{x}{S_{i,max}g_i(r_i)}), \end{aligned}$$

where $F_{\chi_i}(x)$ is the cumulative density function of the i^{th} user's channel power fade. The $\{\chi_i\}$ are assumed to be independent and identically distributed. Differentiating $F_{P_{max}}(x)$ with respect to x gives us the pdf of P_{max}:

$$p_{P_{max}}(x) = \frac{d}{dx}F_{P_{max}}(x) \quad (42)$$

$$= \sum_{i\in I} p_{\chi_i}(\frac{x}{S_{i,max}g_i(r_i)})\frac{1}{S_{i,max}g_i(r_i)} \prod_{j\in I-\{i\}} F_{\chi_j}(\frac{x}{S_{j,max}g_j(r_j)}). \quad (43)$$

We evaluate this distribution for the symmetric case, $S_{i,max}g_i(r_i) = S_{max}$, $\forall i \in I$ assuming Rayleigh fading. The χ_is are exponential distributed with mean Ω, so that

$$p_{\chi_i}(x) = \frac{1}{\Omega}\exp(-\frac{x}{\Omega}) \quad (44)$$

and

$$F_{\chi_i}(x) = 1 - \exp(-\frac{x}{\Omega}). \quad (45)$$

Thus

$$p_{P_{max}}(x) = \frac{K}{\Omega}(1 - \exp(-\frac{x}{\Omega}))^{K-1} \exp(-\frac{x}{\Omega}). \quad (46)$$

The upper bound on average throughput is therefore given as

$$\begin{aligned}
\overline{T} &= \int T_{opt}(\overline{\chi}, \overline{r} | c_1', c_2) p_{P_{max}}(x) dx & (47)\\
&= \sum_{i=0}^{K-1} \binom{K-1}{i} \frac{K}{\Omega} (-1)^i (I_1 + I_2), & (48)
\end{aligned}$$

where

$$\begin{aligned}
I_1 &= D \int_0^C \frac{Kx}{(K-1)x + C} \exp(-\frac{x(i+1)}{\Omega}) dx & (49)\\
&= \frac{DK}{K-1} \left\{ \frac{\Omega}{i+1} \left(1 - \exp[-\frac{C(i+1)}{\Omega}] \right) \right.\\
&\quad \left. - \frac{C}{K-1} \exp[\frac{(i+1)C}{(K-1)\Omega}] \left(\Gamma\left[0, \frac{C(i+1)}{\Omega(K-1)}\right] - \Gamma\left[0, \frac{KC(i+1)}{(K-1)\Omega}\right] \right) \right\}, & (50)\\
I_2 &= \frac{D}{C} \int_C^\infty x \exp[-\frac{x(i+1)}{\Omega}] dx & (51)\\
&= \frac{D}{C} \left(\frac{\Omega}{i+1} \right)^2 \Gamma\left[2, \frac{C(i+1)}{\Omega}\right], & (52)
\end{aligned}$$

and $\Gamma[\alpha, z] = \int_z^\infty t^{\alpha-1} \exp(-t) dt$ is the incomplete gamma function. In Section IX we compare the average throughput given by this upper bound with the optimal average throughput found through Monte Carlo simulations.

VII. Optimum Limited Discrete Rate and Power Adaptation

We now present the optimal rate and power adaptation scheme for an adaptive multiple processing gain CDMA system with discrete, limited rates. The rates transmitted by each user are constrained to a finite, discrete set of possible values. Thus the maximum throughput is given by $T_{opt}(\overline{\chi}, \overline{r} | c_1, c_2, c_3', c_4')$. We start with the following proposition:

Proposition 5: The received powers required to achieve a rate vector $\overline{n} = \{n_1, n_2, \cdots, n_k\}$, $n_i \in Z^+$, can be expressed as

$$P_i(\overline{\chi}, \overline{r}) = C \left(\frac{n_i}{n_i + D} \right) \frac{1}{1-\gamma} \quad \forall i \in I, \tag{53}$$

where $\gamma = \sum_{j \in I} \frac{n_j}{n_j + D}$. Moreover, for a given rate vector to be achievable, we must have $\gamma < 1$.

Proof: Equation (8) can be rewritten as

$$\frac{P_i(\overline{\chi}, \overline{r})}{n_i} = \sum_{j \in I - \{i\}} \frac{P_j(\overline{\chi}, \overline{r})}{D} + \frac{C}{D}, \quad \forall i \in I.$$

Rearranging terms, we obtain

$$P_i(\overline{\chi}, \overline{r}) = \left(\sum_{j\in I} P_j(\overline{\chi}, \overline{r}) + C\right)\left(\frac{n_i}{n_i + D}\right), \quad \forall i \in I \tag{54}$$

$$\Rightarrow \sum_{i\in I} P_i(\overline{\chi}, \overline{r}) = \left(\sum_{j\in I} P_j(\overline{\chi}, \overline{r}) + C\right)\left(\sum_{i\in I}\frac{n_i}{n_i + D}\right), \quad \forall i \in I \tag{55}$$

$$\Rightarrow \sum_{i\in I} P_i(\overline{\chi}, \overline{r}) = C\frac{\gamma}{1-\gamma}. \tag{56}$$

Substituting into (54) we obtain (53). The achievability condition follows since transmit powers must be positive. Note that a rate vector is unachievable if it cannot be achieved for *any* possible fade vector. ■

The achievability constraint $\gamma < 1$ gives us the maximum number of users that can transmit simultaneously as

$$K_{max} < 1 + D. \tag{57}$$

The maximum achievable instantaneous throughput can be found by observing that for a given total throughput, MAI is minimized by allowing the maximum number of users to transmit at the peak rate M. Therefore for our system

$$\max_{\overline{\chi}, \overline{r}} T(\overline{\chi}, \overline{r}|c_1, c_2, c_3', c_4') = \lfloor \frac{M + D}{M} \rfloor M + \lfloor \frac{D(D - M(K-1))}{kM} \rfloor, \tag{58}$$

where $\lfloor x \rfloor$ is the greatest integer *less than or equal to* x.

Also, using the peak power constraint we find that for the rate vector to be achievable with a given fade vector the following must be true:

$$g_i(r_i)\chi_i \geq \frac{C}{S_{i,max}}\left(\frac{n_i}{n_i + D}\right)\frac{1}{1-\gamma} \quad \forall i \in I.$$

This leads to the optimum rate and power adaptation scheme as follows. For every achievable rate vector $\overline{n}$ let us define a vector $\overline{\xi}$ such that $\xi_i = \left(C\frac{n_i}{n_i+D}\right)\frac{1}{1-\gamma}$, and a throughput $T(\overline{n}) = \sum_{i\in I} n_i$. Further, let us arrange these rate and ξ vectors into a table in increasing order of the corresponding throughputs. Table 1 illustrates such a table, where $\overline{\xi}(l)$ denotes the l^{th} ξ vector in the array, $T(l)$ denotes the corresponding throughput, and $\overline{n}(l)$ denotes the corresponding rate vector. Then the optimum throughput, rate, and power for any given fade vector $\overline{\chi}$ and user location vector $\overline{r}$ can be found through a table search as

$$T_{opt}(\overline{\chi}, \overline{r}|c_1, c_2, c_3', c_4') = T(l_{opt}), \quad \overline{n_{opt}}(\overline{\chi}, \overline{r}) = \overline{n}(l_{opt}), P_i(\overline{\chi}, \overline{r}) = \frac{\xi_i(l_{opt})}{\chi_i}S_{i,max},$$

TABLE I

SORTED RATE AND FADE VECTORS WITH $T(l) \leq T(m), \forall\ l < m$

l	$\overline{n}(l)$	$\overline{\xi}(l)$	$T(l)$
1	$\{n_1(1), n_2(1), \cdots, n_K(1)\}$	$\{\xi_1(1), \xi_2(1), \cdots, \xi_K(1)\}$	$\sum n_i(1)$
2	$\{n_1(2), n_2(2), \cdots, n_K(2)\}$	$\{\xi_1(2), \xi_2(2), \cdots, \xi_K(2)\}$	$\sum n_i(2)$
3	$\{n_1(3), n_2(3), \cdots, n_K(3)\}$	$\{\xi_1(3), \xi_2(3), \cdots, \xi_K(3)\}$	$\sum n_i(3)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

where

$$l_{opt} = \max\{l : \chi_j g_j(r_j) > \xi_j(l),\ \forall j \in I\}.$$

Although the size of the table can be large, note that it is much less than M^K since the maximum achievable throughput is much less than MK for large K. Moreover, the size of the table does not increase as the number of users increases beyond K_{max} (defined in (57)). The search in the table can be made efficient in several ways. The ξ vectors, always sorted as $\xi_i(j) \geq \xi_{i+1}(j)$, can also be sorted further in a lexicographic manner *within* the set of ξ vectors that correspond to the same throughput. Also, one can make use of the fact that a minimum throughput of $\sum_{i\in I}\lfloor\frac{P_{i,max}(\chi_i,r_i)D}{\sum_{j\in I-\{i\}}P_{j,max}(\chi_j,r_j)+C}\rfloor$ is obviously achievable, and therefore search for only higher throughputs. And of course, it is possible to have a tradeoff between optimality and complexity by limiting the complexity of the search.

It is also worth mentioning here that although the peak *received* power $P_{i,max}(\chi_i, r_i)$ of a user changes over time, the table uses only the peak *transmit* power $S_{i,max}$ which does not change.

VIII. Optimum Power Control versus Optimal Rate Control

In this section we wish to find out the maximum average throughputs achievable with rate or power control alone. Recall that with adaptive modulation in narrowband systems it was found that optimal rate control yields almost all the throughput achievable with both rate and power adaptation. In particular it performs significantly better than power adaptation alone [7]. In this section, we wish to perform a similar comparison for adaptive multirate CDMA. First we consider optimum power control. We restrict our system so that whenever a user transmits, he uses a fixed rate M, while the transmit power is adapted to the channel fade vector. The average throughput for such a system is

$$\overline{T} = M\mathrm{E}[k(\overline{\chi})], \tag{59}$$

where $k(\overline{\chi})$ is the number of users transmitting at rate M for a given channel fade vector $\overline{\chi}$. Note that since we are interested in the average over the fading distribution, we consider the symmetric case, where all users have the same propagation path loss which can be incorporated into the peak power constraint, so that $P_i(\overline{\chi}, \overline{r}) = P_i(\overline{\chi}) \leq S_{i,max} g_i(r_i)\chi_i = S'_{max}\chi_i$. Now, if $k(\overline{\chi})$ users are transmitting at the same rate simultaneously, they must all have the same received power $P_k(\overline{\chi})$ such that

$$\frac{DP_k(\overline{\chi})}{(k(\overline{\chi})-1)P_k(\overline{\chi})+C} = M, \quad (60)$$

$$\Rightarrow P_k(\overline{\chi}) = \frac{C}{\frac{D}{M}+1-k(\overline{\chi})}. \quad (61)$$

This implies that

$$\chi_i > \frac{C}{S_{max}(\frac{D}{M}+1-k(\overline{\chi}))} = \chi_{min}(k) \quad \forall i \in I_{trans}(\overline{\chi}), \quad (62)$$

where $I_{trans}(\overline{\chi})$ is the index set of transmitting users and $k(\overline{\chi}) = |I_{trans}(\overline{\chi})|$. The optimum power adaptation in this case is to choose the maximum k such that at least k users have channel fades better than $\chi_{min}(k)$. The optimum powers of these k users are given by (61).

The probability that at least k users have channel fades better than $\chi_{min}(k)$ is given by:

$$\text{Prob}[k(\overline{\chi}) \geq k] = \text{Prob[k or more users have fades}\chi_i \geq \chi_{min}(k)] \quad (63)$$

$$= \sum_{i=k}^{K} \binom{K}{i} p_k^i (1-p_k)^{K-i}, \quad (64)$$

where $p_k = \text{Prob}[\chi_i > \chi_{min}(k)]$ and the $\{\chi_i\}$ are independent, identically distributed. Thus we have

$$\overline{T} = M\text{E}\{k(\overline{\chi})\}, \quad (65)$$

$$= M \sum_{k=1}^{\min(K,\lfloor D/M+1 \rfloor)} k\text{Prob}[k(\overline{\chi}) = k], \quad (66)$$

$$= M \sum_{k=1}^{\min(K,\lfloor D/M+1 \rfloor)} k\left(\text{Prob}[k(\overline{\chi}) \geq k] - \text{Prob}[k(\overline{\chi}) \geq k+1]\right), \quad (67)$$

$$= M \sum_{k=1}^{\min(K,\lfloor D/M+1 \rfloor)} k\left(\sum_{i=k}^{K} \binom{K}{i} p_k^i (1-p_k)^{K-i} - \sum_{i=k+1}^{K} \binom{K}{i} p_{k+1}^i (1-p_{k+1})^{K-i}\right). \quad (68)$$

This gives us the maximum average throughput achievable with power control, given a fixed transmit rate M for every user. The throughput is still a function of M and an optimum value of M can be chosen for a given K. In Section IX, we evaluate the maximum average throughput achieved with optimum power control and find the optimum M (assuming unrestricted rates) for different K.

Next we look at optimum rate adaptation. We restrict our system so that whenever a user transmits, he uses a fixed power P, while the rate is adapted to the channel fade vector. We assume the rates to be continuous and unlimited. But this gives us our optimum power and rate adaptation scheme considered in Section IV. The optimum rate and power adaptation with unconstrained rates required that a user, whenever he transmits, uses his peak power. Hence, the optimum rate and power adaptation is actually just the optimum rate adaptation.

IX. Numerical Results

In this section we present the results of simulations carried out in order to test the algorithms developed in the previous sections and to gain further insight into the throughput achievable with adaptive multirate CDMA. The average throughput under different constraints is found and plotted in Figure 1. The parameters common to the curves in Figure 1 are a maximum spreading gain of $N = 64, D = \frac{3}{2(E_b/N_e)_o} = 20$, and $\left(\frac{S_{max} g(r) N T_c}{N_o}\right) = 12$db. All users are assumed to have the same peak transmit power and the same propagation path loss. Channel power fades are assumed to be exponential distributed (flat Rayleigh fading) with unit variance. While we use flat fading for simplicity, note that the algorithms developed earlier can also be used with frequency selective fading as explained in Section II. Limited rates schemes allow users to transmit at rates up to 1/6. With these parameters, the maximum number of users that can transmit simultaneously is $K_{max} = 20$. Average throughput for the various schemes is found using Monte Carlo simulations. The analytical upper bound curve is also plotted in the same figure. As expected, for a given number of users in the system, the average throughputs for various schemes are in the order:

Analytical Upper bound (AU)> Optimum Unlimited Continuous Rate and Power Adaptation (OUCRPA) > Optimum Limited Continuous Rate and Power Adaptation (OLCRPA) > Optimum Limited Discrete Rate and Power Adaptation (OLDRPA) > Optimum Power Adaptation (OPA).

The optimum power adaptation (OPA) curve is plotted for transmit rate M=1/32. The dependence of maximum average throughput on M for various K is shown in Figure 2. From Figure 2 we observe that for a given K, there is an optimum value of M that maximizes the average throughput. Choosing M to be higher than this value reduces the average throughput. Also, as the number of users increases lower values of M are found to be optimal. Going back to Figure 1, we note that the average throughput achieved with optimum power adaptation (OPA) is significantly lower than that obtained with optimum rate adaptation(OUCRPA).

The curves for OUCRPA and OLCRPA suggest that limiting the maximum achievable instantaneous rate for a user does not significantly reduce the average throughput. This can be explained as follows. In order to transmit a higher rate, a user needs a better channel, since his power is limited and he needs a minimum SNR. However, for most fading distributions the probability of a better channel (higher channel power gain) decreases exponentially. So the contribution to average throughput from higher rates is reduced correspondingly.

The unlimited continuous rate curves are useful as a theoretical upper bound for the performance of all schemes. They also serve as estimates for the curve of greatest interest to practical (finite, limited rates) systems, i.e. the OLDRPA. Note that with i.i.d. users, as the number of users in the system goes beyond K_{max} the maximum average throughput keeps increasing because the *best* (strongest) users keep getting *better* (stronger). For large K_{max} however, the size of the table required for ODLRPA grows exponentially, increasing the complexity of the search for the optimum rate vector. For this reason, we now look at other schemes that can achieve close to optimal performance with a much lower complexity.

The *Discrete Unlimited Rate and Power Adaptation*(DURPA) curve is obtained by adapting the OUCRPA scheme to discrete rates. The adaptation is such that

$$T(\overline{\chi}, \overline{r}) = \max_{n \in I} D \sum_{i \in I_{opt}} \left\lfloor \frac{P_{i,max}(\chi_i, r_i)}{\sum_{k \in I_{opt}-\{i\}} P_{k,max}(\chi_k, r_k) + C} \right\rfloor \tag{69}$$

where $I_{opt} = \{1, 2, \cdots, n\}$. As apparent from Figure 1, this scheme provides a good approximation to ODLRPA. The *Quantized Unlimited Rates and Power Adaptation*(QURPA) curve represents a scheme that is obviously far from desirable. It is included in Figure 1 to show that a "direct" quantization of OUCRPA fails miserably. By direct quantization we mean that for every given fade vector $\overline{\chi}$ and location vector $\overline{r}$, we find the OUCRPA rate vector $\overline{n}, n_i \in R^+$ and let every user transmit at rate $\frac{1}{\lceil 1/n_i \rceil}$. That is, the optimum rate vector is quantized by component-wise truncation. Obviously, as the number of transmitting users increases, more quantizations have to be performed and the throughput actually *decreases.* The last remaining curve in the figure corresponds to a *Limited Discrete Rate* suboptimal scheme that does not use a table-lookup. Instead it makes use of an initial conservative estimate of achievable throughput provided by QURPA, and then tries to increase the rates of the *best* users, since the users with the best channels are most likely to be capable of supporting higher rates. The complexity of this suboptimal scheme is $O(K^2)$, and the throughput curve suggests that it performs reasonably well.

As stated earlier, the OUCRPA and OLCRPA curves in Figure 1 seem to suggest that increasing the total number of rates assigned to each user above a significantly large value does not significantly increase throughput. Figure 3 also verifies this result: The gains from having a bigger set of rates for each user quickly get saturated. Figure 4 shows hows P_{max} affects the system. The law of diminishing returns is evident in this curve.

Note that in this paper our aim is maximize the total throughput without any fairness constraints. We therefore expect that the solutions obtained in this manner will allocate higher rates to users located closer to the base station who suffer less path loss than the users located farther away. This is an example of the well known near-far problem. The total throughput, which is our focus in this paper, is not a good indicator of the fairness of the solution. A full-fledged simulation of a practical system using the algorithms developed here, with particular attention to the individual rates achieved by the various users, would provide a realistic estimate of the severity of this problem and point towards additional constraints that need to be imposed to ensure fairness across users.

X. Conclusion

We derive the maximum throughput achievable in a variable rate variable power CDMA system using a conventional matched filter receiver and a target bit energy-to-equivalent noise spectral density ratio $(E_b/N_e)_o$. We start with a multiple processing gain CDMA system with unconstrained rates available to each user. We show that in this case for a given channel fade vector the optimum rates and powers can be found with computational complexity $\sim O(K^2)$, where K is the number of users. The loss of throughput due to discrete, bounded rates and an analytical upper bound on the average throughput with unconstrained rates is also derived. We then consider practical multiple processing gain CDMA systems that restrict the values of the users' bit duration to a finite number of integer multiples of chip duration. We show that in this case the optimum powers and rates for a given fade vector can be found through a table search. Numerical results for the average throughputs in all cases are also presented.

We find that the optimum rate and power adaptation scheme with unconstrained rates is in fact just a rate adaptation scheme, with fixed transmit powers. On the other hand, the optimum power adaptation scheme with fixed rates yields significantly lower average throughput. Thus although power adaptation is much simpler to implement than rate adaptation, it can never achieve the average throughputs possible with rate adaptation.

We also find that increasing the maximum rate available to each user beyond a given value does not significantly increase throughput. Similarly, restricting the rates of the adaptive multiple processing gain system to discrete values does not significantly decrease throughput. Similar results were obtained for narrowband single user systems in [7]. We also analyze some suboptimal schemes with lower complexity to determine the performance tradeoffs between complexity and optimality. We observe that it is possible to reduce the complexity of power and rate adaptation for a multiple processing gain system with discrete, finite rates to $\sim O(K^2)$ without significantly decreasing throughput. We also find that a peak power constraint limits the maximum achievable average throughput even with unlimited rates available to every user. This is in contrast to the case where there is an average power constraint: in this case it can be shown that the unlimited rates assumption leads to unbounded average throughput. Thus, a maximum power constraint is a more fundamental limitation than an average power constraint.

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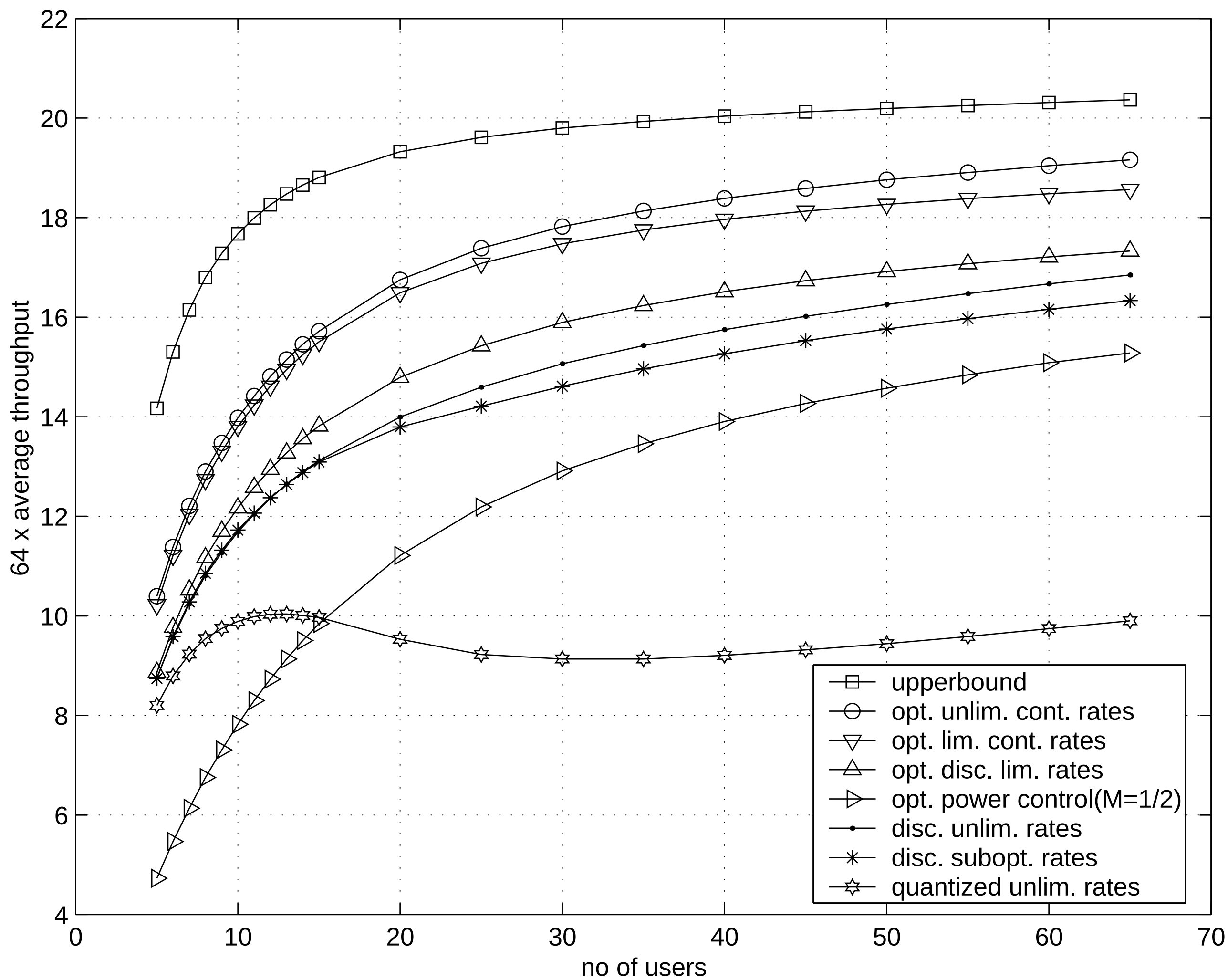

Fig. 1. Average throughputs for different schemes

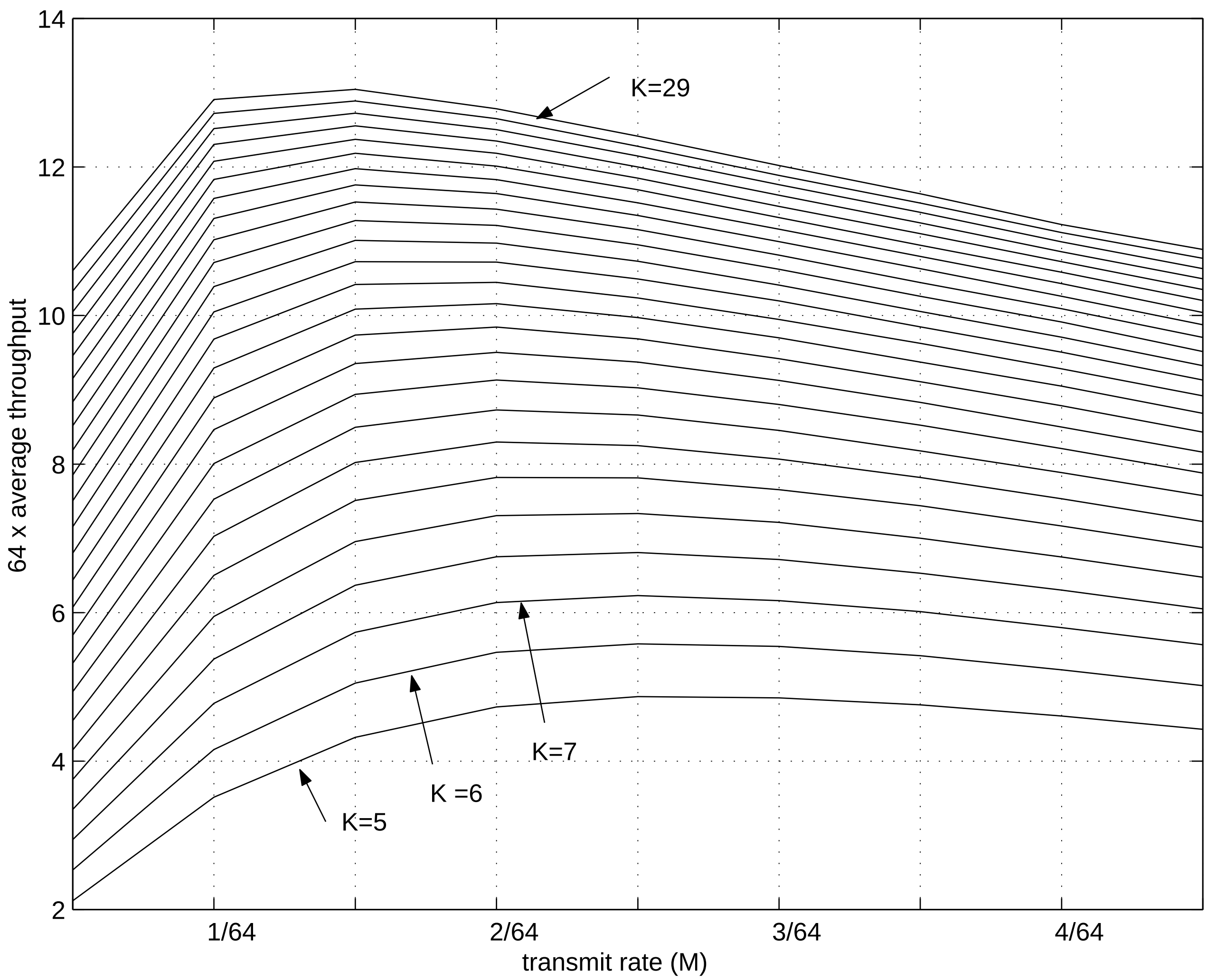

Fig. 2. optimum power control

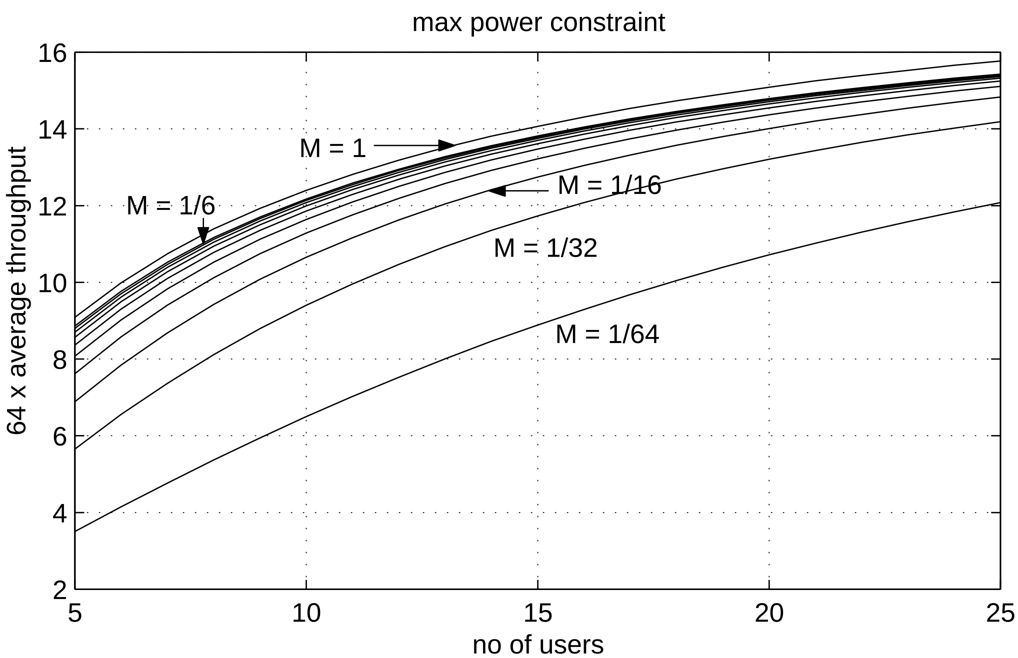

Fig. 3. Effect of max. rate M

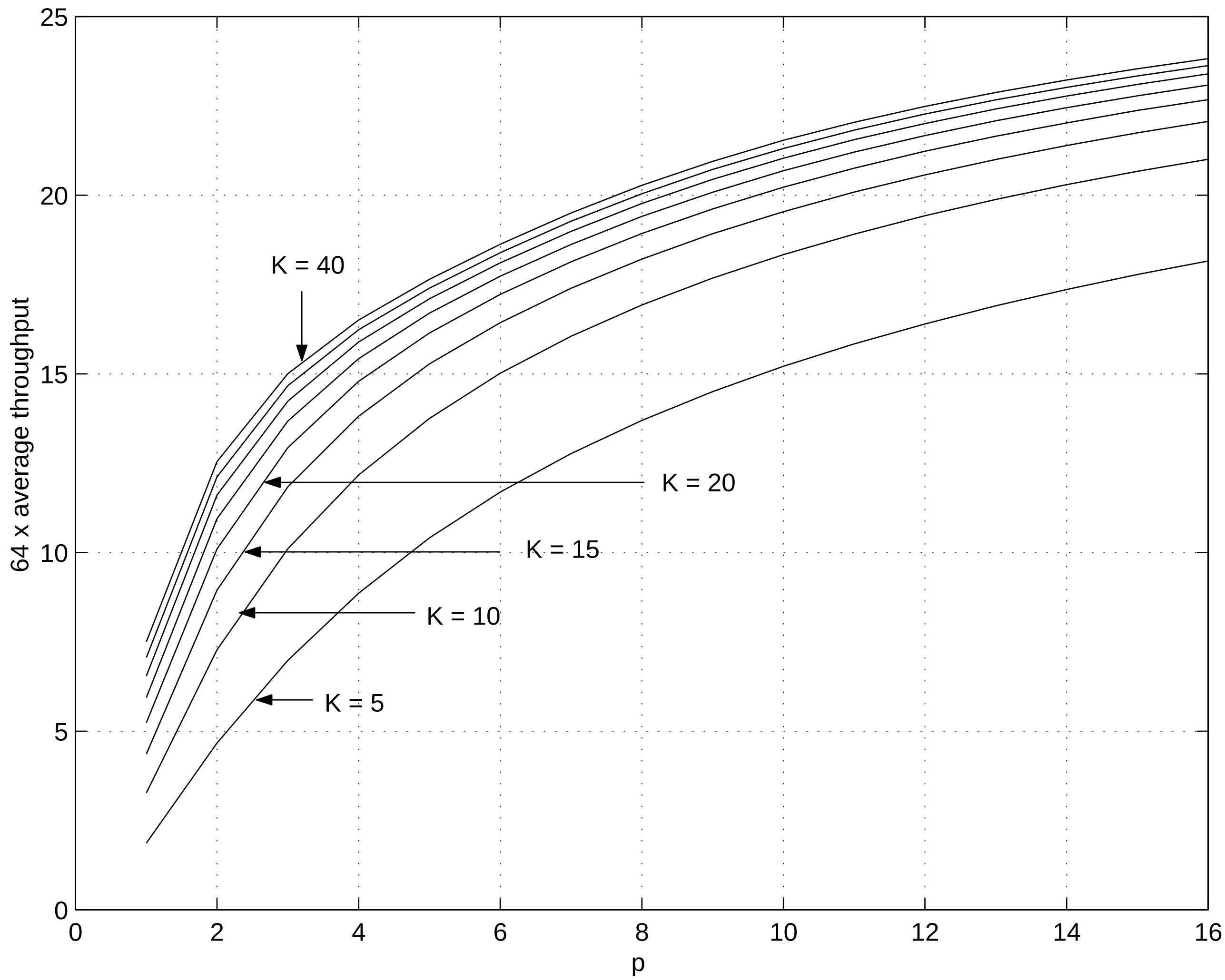

Fig. 4. Average throughput versus Peak Power $\left(\frac{P_{max}NT_c}{No} = \frac{p}{4}G,\ 10\log G = 12\ db\right)$