PhantomNet: Exploring Optimal Multicellular Multiple Antenna Systems

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***Abstract*— We address the problem of providing the best possible service to new users joining a multicellular multiple antenna system without affecting existing users. Since, interferencewise, new users are invisible to existing users, the network is dubbed Phantom Net.**

I. Introduction

A traditional way of handling the multiantenna, multiuser and multicellular system has been to reduce it to a single antenna, single user, single cell system by orthogonally splitting the channel among the users in time/frequency/code/space, employing the base station antennas for sectoring/beamforming, and treating co-channel interference from other cells as noise. Moreover, since early wireless networks have been designed primarily for voice traffic, rate adaptation was not considered. This constrained approach may be simpler, but quite often it leads to quite suboptimal strategies. In order to estimate the absolute performance limits of these multidimensional systems we need to explicitly account for the presence of multiple users, multiple antennas and multiple cells on both the uplink and the downlink.

In this paper, where wireless data communication is highlighted, the focus is on finding the best transmit strategy. The best strategy, of course, depends on the priorities assigned to each user. Given the prioritization, say, e.g., First Come First Served (FCFS), we find here the optimum communication means under different criteria.

Although we will proceed with the FCFS prioritization in our presentation, our results hold for other means of prioritizing like: last-come-first-served, random ordering, or any scheme that predetermines an ordering among users.

We consider both the uplink and the downlink of a multiuser multicellular system using multiple antennas at both ends. We consider a system that evolves in time with new users entering the system and old users leaving the system. Using FCFS, our objective is to provide the best service possible to the new users as they enter the system, without penalizing the users already in the system. Thus each user in the system has a higher priority than the users that come after him. Subsequent users are served under the requirement that the previous ones are not affected: interferencewise, new users must be invisible to existing users. Since for both the uplink and the downlink only earlier entrants interfere while later entrants are invisible, the network is dubbed Phantom Net. The strategies that effect this invisiblity will be seen to be Successive Decoding (SD) for the uplink and Known Interference (KI) coding for the downlink. (These are forms of, so called, Multiuser Detection and Dirty Paper Coding, respectively.) In our network context, these strategies are particularly interesting both because of their simplicity as well as the unmistakable symmetry evident between uplink downlink operation.

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II. System Model

Although we are ultimately interested in a multicellular system, for simplicity, we start with a single base station. Multiple base stations will be addressed in Section VI.

A. Uplink

The uplink is characterized by the equation $Y = \sum_{i=1}^{K} H_i X_i + N$. where Y is the received vector at the base station, K is the number of users currently active in the system, H_i is the flat fading matrix channel of user i, and N is the Additive White Gaussian Noise (AWGN) vector at the base station.

Without loss of generality we assume that the users are indexed by the order in which they arrive. So user 1 is the first user in the system while user K is the last user to join the system. The users are subject to transmit power constraints given by trace $\left[\mathrm{E}[X_i X_i^\dagger]\right] \le P_i, \quad 1 \le i \le K$.

B. Downlink

Finding the optimal transmit strategy for the downlink with multiple antennas is a hard problem. This is because the multiple antenna downlink channel is a non-degraded broadcast channel and its capacity region is a long standing unsolved problem in information theory [1]. The optimal coding strategy for the multiple antenna downlink is therefore unknown. The special cases of the AWGN broadcast channel where the capacity and the optimal coding strategy are known include the degraded broadcast channel (single transmit antenna at the BS), and the recently solved sum capacity point of the multiple user multiple antenna channel [2][3]. While SD achieves capacity in the first case, KI coding based on the results of [4] achieves capacity in the latter. KI coding can also be shown to achieve capacity for the degraded AWGN broadcast channel. For this reason we restrict our downlink transmit strategy to these two coding schemes. This is reflected in our downlink model, given by the equation: $Y_i = H_i \sum_{j=1}^{K} X_j + N_i$, where Y_i, X_i, H_i, and N_i are the output vector, the input vector, the channel matrix, and the AWGN vector for user i. Note that for both SD and KI coding strategies, the input vectors corresponding to different users are independent. As in the uplink model described earlier, the downlink model also assumes that the users are indexed by the order in which they arrive. Further, the power in each user's input vector is given by trace $\left[\mathrm{E}[X_i X_i^\dagger]\right] \le P_i, \quad 1 \le i \le K$.

For both the uplink and the downlink the channel is assumed to experience slow and flat fading. We assume that the channel matrices are perfectly known to the BS. The users are assumed to know their own channel and the spatial covariance structure of the sum of the noise and the relevant interference seen at the receiver.

III. Problem Definition

Based on the first-come-first-served model, our primary objective is to accommodate new users only to the extent that the users that are already active in the system are not affected. While this constitutes the general idea, to be precise we need to distinguish between the following two cases.

Existing Users are Unaffected (Preserving Rates): This would mean that the existing users continue to have the same rates as before. However, this leaves open the possibility that the existing users may adjust their transmit strategy on the uplink or their receive strategy on the downlink in some way to accommodate the new user. For example, on the downlink, it is conceivable that if superposition coding was used then the existing users may need to decode and subtract out the new users signal before detecting their own signal. If this allows the existing users to achieve the same rates as before, we say that the existing users are not affected, or, the rates are preserved.

Existing users are Strictly Unaffected (Making the Accommodation of New Users Invisible): We could be more strict in our problem statement. We could demand that the new users be accommodated in such a way that not only do the existing users continue to achieve the same rates as before but they are completely oblivious to the presence of new users. That is, the existing users' transmitters/receivers on the uplink/downlink continue to process the input data stream/received signal exactly as before to generate the transmitted signal/output data stream. Thus the only changes needed to accommodate the new user are made at the base stations. To distinguish this case from the previous one, we say that the existing users are *strictly* unaffected, or, the new users are invisible.

Within each of the cases mentioned above, there are several, more or less equally significant, problems that one can pose. We list these problems in the following subsections III-A and III-B for the uplink and the downlink respectively. We will see later that all the uplink problems really amount to the same problem - basically the same solution procedure covers all of the uplink variations. Among the downlink problems we will encounter some substantive differences.

A. Uplink

On the uplink the user's transmit power is the limiting factor. So, for the uplink, the first set of problems UP1a and UP1b (Uplink Problem 1a and 1b) that we wish to solve are:

- **UP1a:** *(Preserving Rates)* Allocate the maximum possible rate to user K (new user) with transmit power P_K such that the existing users' rates are not affected.
- **UP1b:** *(Making the New User Invisible)* Allocate the maximum possible rate to user K (new user) with transmit power P_K such that the existing users are strictly unaffected.

B. Downlink

On the downlink each base station distributes the total transmit power among the users it serves. Thus, unlike the uplink where each user has an individual power constraint, the downlink is characterized by a sum power constraint instead. The coding schemes we consider for the downlink are SD and KI. A brief description of these schemes is presented later. In particular we wish to determine the following:

- **DP1:** Is KI or SD a better scheme for the downlink?

For FCFS scheduling, the corresponding problems on the downlink would be:

- **DP2a:** *(Preserving Rates)* Determine the maximum possible rate for user K subject to a total transmit power $P_1 + P_2 + \cdots + P_K$, such that existing users' rates are not affected.
- **DP2b:** *(Making the New User Invisible)* Determine the maximum possible rate for user K subject to a total transmit power $P_1 + P_2 + \cdots + P_K$, such that existing users are strictly unaffected.

Note that in problems DP2a and DP2b, the BS adds a power P_K to the total power to accommodate a new user (user K) into the system. The powers $P_1, P_2, \cdots, P_K$ determine how the rates are allocated to the users and need not be the actual transmitted powers in each user's input signal.

Note that as the channel changes, the users' rates/powers may change. So for each channel realization we solve the FCFS scheduling problems listed above. The assumption that the channel varies slowly is important in this respect.

IV. Uplink Solution

The uplink presents a relatively simple problem since the capacity region and the optimal coding strategy are known even with multiple antennas at the BS and the mobiles[5]. The desired solution is therefore easily seen to be as follows.

The solution to the first uplink problem **UP1a** (Preserving Rates) is given by the following proposition.

Proposition 1: The optimal set of rates $R_i^\star$ on the uplink are

$$R_i^\star = \log \left| I + \left(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger \right)^{-1} H_i Q_i^\star H_i^\dagger \right| \quad (1)$$

where $Q_i^\star$ is the optimal input covariance matrix obtained by waterfilling over the eigenmodes of the effective channel matrix $(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger)^{-\frac{1}{2}} H_i$ subject to the power constraint trace$(Q_i) = P_i$.

In other words, the optimal strategy for the uplink is to use Successive Decoding (Multiuser Detection with Successive Interference Cancellation) at the base station in the inverse order of the user's indices. The new user gets decoded first and his signal is subtracted out so that the existing users do not see him as interference. The highest rate that the new user can support without affecting existing users is simply given by the single user waterfilling solution treating the existing users' signal as colored Gaussian noise.

It is interesting to note the simplicity of the solution. Note that the SD scheme requires only the BS to make some changes in the way it decodes the received signal. Specifically, the BS needs to decode the new user and subtract his signal before proceeding to decode the existing users' signals. However, the existing users themselves do not need to do anything different because of the new user. Thus the new user is completely invisible to existing users. Thus, we conclude that on the uplink the optimal strategy that leaves the existing users' rates unaffected also leaves the existing users strictly unaffected. In particular the optimal solution to UP1a (Preserving Rates) is also the optimal solution to UP1b (Making the New User Invisible).

V. DOWNLINK

A. Solution to DP1 (KI versus SD)

With the following proposition we show that KI is the better downlink strategy in general.

Proposition 2: Subject to a sum power constraint, the set of rate vectors achievable with SD and time sharing is also achievable with KI and time sharing.

Proof: We prove this by showing that the boundary of the achievable rate region with SD and time division is contained within the boundary of the achievable rate region with KI and time sharing. Note that in either scheme the points in the interior can always be attained by throwing away some codewords.

The boundary points of the rate region are obtained by maximizing $\sum_{i=1}^{K} \mu_i R_i$ for all $\vec{\mu}$ such that $\vec{\mu} \geq \vec{0}$ and $\sum_{i=1}^{K} \mu_i = 1$. Let $\mathcal{R}^{SD}$ and $\mathcal{R}^{KI}$ denote the sets of rate vectors achievable with SD and KI respectively. Note that in order to prove the result of Proposition 2 it suffices to prove that for all $\vec{\mu}$,

$$\max_{\vec{R} \in \mathcal{R}^{KI}} \sum_{i=1}^{K} \mu_i R_i \geq \max_{\vec{R} \in \mathcal{R}^{SD}} \sum_{i=1}^{K} \mu_i R_i. \quad (2)$$

In order to prove (2) we assume without loss of generality that the users' priorities are arranged as $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_K$. We start with the SD scheme and show that KI can achieve at least the same value of $\vec{\mu} \cdot \vec{R}$. Let $\vec{R}^{SD}$ be the rate vector that maximizes $\vec{\mu} \cdot \vec{R}$ with SD. Without loss of generality we can assume that $\vec{R}^{SD}$ does not use timesharing. This is because simple linear programming tells us that a rate vector corresponding to timesharing between several different rate vectors is a convex combination of those rate vectors and therefore can not achieve a higher value of $\vec{\mu} \cdot \vec{R}^{SD}$ than the best of those rate vectors.

Let the total number of substreams being transmitted be L. Further, and again without loss of generality, let us label the substreams from 1 to L such that if $i < j$ and substream i carries data for user $u(i)$ and substream j carries data for user $u(j)$ then $\mu_{u(i)} \geq \mu_{u(j)}$. That is, the substreams are arranged in decreasing order of the priority of the user whose data they are carrying. For multiple substreams carrying the same user's data, we label them in the order in which they are decoded by that user.

Now note that no user can decode a substream carrying data for a user with a lower priority. This is easily proved by contradiction as follows. Suppose user A can decode a substream that carries user B's data at a rate r. Now if user A has a higher priority than user B, i.e. if $\mu_A > \mu_B$ then we can increase $\vec{\mu} \cdot \vec{R}^{SD}$ by simply having the substream carry user A's data instead of user B's data, at the same rate r. So that,

$$\vec{\mu} \cdot \vec{R}(\text{new}) = \vec{\mu} \cdot \vec{R}^{SD} - \mu_B r + \mu_A r > \vec{\mu} \cdot \vec{R}^{SD}. \quad (3)$$

But this is a contradiction since we assumed that the rate vector $\vec{R}^{SD}$ maximizes $\vec{\mu} \cdot \vec{R}$ over all rate vectors $\vec{R}$ achievable with SD and without timesharing.

In light of this observation, it is clear that while decoding substream l, the intended user must treat substreams $l+1$ to L as noise. The substreams 1 to $l-1$ may or may not be treated as noise depending upon whether it is possible to decode and subtract those substreams or not. So with SD, the rate achieved on the l^{th} substream is no greater (could be smaller) than r_l, where r_l is the achievable rate when the substreams $l+1$ to L are treated as noise while substreams 1 to $l-1$ are not present. Next we show that KI can achieve r_l on each of these substreams.

Suppose we use KI to encode the L substreams in the order in which they are labeled. Then the l^{th} substream sees substreams $l+1$ to L as noise since these substreams are encoded *after* substream l and therefore the interference caused by them is not known. However, since substreams 1 to $l-1$ have already been encoded they present known interference to substream l and therefore do not affect the data rate that substream l is capable of supporting. Thus KI allows substream l a rate r_l that is at least as large as the maximum allowed rate for that substream in the optimum SD rate vector that maximizes $\vec{\mu} \cdot \vec{R}$. This proves (2) and completes the proof of Proposition 2. ■

B. Downlink Solutions for DP2a (Preserving Rates) and DP2b (Making the Accommodation of New Users Invisible)

1) Solution to DP2a (Preserving Rates): In DP2a we are only requiring rate conservation in dealing with the Kth user. This leaves open the possibility, that, in meeting the earlier rates, if the earlier users are handled in a different way than before, we can actually achieve a strictly greater rate for the Kth user. Indeed, in some instances a greater rate is possible. This DP2a problem is exceptional in that, we encounter the most difficult of the optimization problems in this paper. In the general case a solution can, in theory, be obtained by solving a number of convex programming problems to obtain the achievable rate region with Dirty Paper Coding [3]. However, the complexity of this is exponential in the number of users.

2) Solution to DP2b (Making the Accommodation of New Users Invisible):

Proposition 3: The optimal set of rates $R_i^\star$ on the downlink such that existing users are oblivious to the presence of the new users are given by

$$R_i^\star = \log \left| I + \left(I + \sum_{j=1}^{i-1} H_i Q_j^\star H_i^\dagger \right)^{-1} H_i Q_i^\star H_i^\dagger \right| \quad (4)$$

where $Q_i^\star$ is the optimal input covariance matrix obtained by waterfilling over the eigenmodes of the effective channel matrix $(I+\sum_{j=1}^{i-1} H_i Q_j^\star H_i^\dagger)^{-\frac{1}{2}} H_i$ subject to the power constraint trace$(Q_i) = P_i$.

In other words, the optimal strategy for the downlink that does not allow new users to affect existing users is to use KI encoding at the base station in the inverse order of the user's indices. The new user gets encoded first so his signal is known interference and the existing users' rates do not get affected. The highest rate that the new user can support without affecting existing users is simply given by the single user waterfilling solution treating the existing users' signal as colored Gaussian noise.

Proof: KI's ability to handle arbitrarily varying interference [6] makes it the obvious choice in this case. Using SD would require existing users to decode the new user, thus acknowledging the new user's presence. However, since KI is able to handle arbitrary interference, it does not matter if the interference known to the i^{th} user's encoder comes from users $i, i+1, \cdots, K-1$ or from users $i, i+1, \cdots, K$. The rate and decoding strategy for user i depend only on the interference from users $1, 2, \cdots, i-1$ that came before him and whose signals must be treated as noise for user i. ■

The solution for the downlink is interesting for its simplicity and also for its striking symmetry with the uplink solution.

VI. Multiple Base Stations

In this section we incorporate multiple base stations to model a multicell environment. We assume that all the base stations are connected through a high speed reliable network. It allows perfect coordination and information exchange between base stations.

A. Uplink

On the uplink, since we allow perfect coordination and information exchange between base stations, note that we can treat all the base stations together as one big base station with all the antennas. This brings us back to the single cell model. Thus, for the uplink the optimal solutions for the single cell simply carry through to the multicell environment.

B. Downlink

We present the downlink solution to DP2b (existing users oblivious to the presence of new users) with multiple cells. The downlink with B base stations is described as

$$Y_i = \sum_{b=1}^{B} H_i^{[b]} \sum_{j=1}^{K} X_j^{[b]} + N_i, \quad 1 \le i \le K,\ 1 \le b \le B, \quad (5)$$

where Y_i is the output vector, $X_i^{[b]}$ and $H_i[b]$ are the input vector and the channel matrix from base station b, and N_i is the AWGN vector for user i. Further, the additional power for each new user is limited per base station so that,

$$\text{trace}\left[\mathrm{E}[X_i^{[b]} X_i^{[b]\dagger}]\right] \le P_i^{[b]}, \quad 1 \le i \le K,\ 1 \le b \le B. \quad (6)$$

Note that a system where each user is assigned to only one base station is included as a special case by setting the appropriate power constraints to zero.

Again, since we allow perfect coordination between base stations we can represent the B base stations as one composite base station. Defining,

$$H_i = \left[H_i^{[1]}\ H_i^{[2]}\ \cdots\ H_i^{[B]}\right], \quad 1 \le i \le K, \quad (7)$$

and X_i as the vector obtained by stacking all the $X_i^{[b]}$ into a single column, we obtain an equivalent representation for the downlink as $Y_i = H_i \sum_{j=1}^{K} X_j + N_i$. This looks similar to the single cell downlink model we had earlier. However, note that the components of the input vector X_i come from different base stations. There is a different input power constraint on each base station. Thus, the solution presented earlier does not apply in the exact same form.

We explain the new downlink solution in terms of rate-splitting for clarity. Specifically, we split each user's rate into B substreams. The idea is to perform the waterfill in B stages. At each stage, we waterfill until a base station meets its power constraint. Then we null out the antenna gains from that base station so that no more power is allocated to it and proceed with the waterfill. This gives us B layers or B substreams that can be encoded using KI. Consider the i^{th} user. As shown in Proposition 3 this user sees the interference from users $1, 2, \cdots, i-1$ as colored noise and is unaffected by the interference from users $i+1, i+2, \cdots, K$. Therefore the maximum rate he can achieve is given by

$$R_i^\star = \max_{Q_i} \log \left| I + \left(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger\right)^{-1} H_i Q_i H_i^\dagger \right| \quad (8)$$

where the maximization is over all input covariance matrices that satisfy the power constraints per base station. We split the user's rate into B substreams, to be encoded in the order $B, B-1, \cdots, 1$ using KI encoding. So the B^{th} substream sees all the other substreams as noise, while the 1^{st} substream's rate is unaffected by substreams $B, B-1, \cdots, 2$. Let the rates on these substreams be $R_i^{[b]\star}$, and the corresponding input covariance matrices be $Q_i^{[b]\star}$. Then we have

$$R_i^\star = R_i^{[1]\star} + R_i^{[2]\star} + \cdots + R_i^{[B]\star} \quad (9)$$

$$Q_i^\star = Q_i^{[1]\star} + Q_i^{[2]\star} + \cdots + Q_i^{[B]\star} \quad (10)$$

The optimal $Q_i^\star$ is obtained as follows.

1) Perform a singular value decomposition of the effective composite channel $\left(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger\right)^{-\frac{1}{2}} H_i$ as

$$\left(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger\right)^{-\frac{1}{2}} H_i = F_i \Lambda_i M_i. \quad (11)$$

Start water-pouring over the eigenmodes of this channel. Continue adding power until one of the base stations meets its power constraint for the i^{th} user, $P_i^{[b]}$. Without loss of generality we assume base station 1 runs out of power for user i. This corresponds to the first rate split, ie. call this the 1^{st} substream for user i. The input covariance matrix obtained in this way is $Q_i^{[1]\star}$. Among the B substreams corresponding to user i, this substream will be encoded last, so it is unaffected by the interference from the remaining $B-1$ substreams. The rate on this substream is

$$R_i^{[1]\star} = \log \left| I + \left(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger \right)^{-1} H_i Q_i^{[1]\star} H_i^\dagger \right|$$

2) Since base station 1 already used up its power for user i, we null out the contribution from $H_i^{[1]}$ to the compound channel matrix by setting it to zero. Define a new composite channel

$$H_i^{[-1]} = \left[\mathbf{0} \ H_i^{[2]} \ H_i^{[3]} \ \cdots H_i^{[B]} \right]. \qquad (12)$$

Again, perform a singular value decomposition on the new composite effective channel

$$\left(I + \sum_{j=1}^{i-1} H_j Q_j^\star H_j^\dagger + H_i Q_i^{[1]\star} H_i^\dagger \right)^{-\frac{1}{2}} H_i^{[-1]} \qquad (13)$$

Note that this treats the interference from the first substream as noise. Again, start water-pouring over the eigenmodes of this new channel, until another base station meets its power constraint

Proceeding in this fashion we obtain the input covariance matrices on all the substreams and the corresponding rates as well. Combining the substreams we get the overall rate and input covariance matrix for each user from equations (9) and (10).

Thus we find that multiple base stations only affect the downlink solution to the extent that the waterfilling algorithm needs some modification in order to accommodate the different power constraints per base station. Otherwise, the solution does not change. In particular, KI is still the optimal strategy and the ordering of users is the same as before.

VII. Conclusions and Discussion

We addressed the problem of providing best possible rates to new users as they enter a wireless data network, without penalizing the existing users. We have dubbed the network a Phantom Net. This is because of the design theme, that when a user enters, all subsequent entrants must, to him, be phantoms, i.e., inteferencewise they must be invisible. For both the uplink and the downlink, only earlier entrants can interfere with an entering user. Phantom Net operation involves treating all bases as a single composite base, so that the actual bases simply serve as multiple antenna sites which are networked, say with fibers, to and from a single central processor.

For the uplink we found that, to achieve the phantom requirement, we could make a straightforward application of the well established Successive Decoding strategy where the new user is *decoded* before the existing users. For the downlink, achieving the invisibility requirement is more problematic. The optimal downlink strategy is to use Known Interference Coding where the new user is *encoded* before the existing users. This makes use of the fact that the bases have knowledge of all signals that are to be transmitted. This enables simultaneous communication to the users despite arbitrarily varying interference by signalling modulo a fundamental lattice cell and using dithering techniques.

The striking feature of the uplink and the downlink strategies is their simplicity, and even more than that, their similarity. In both cases, the new users are forced to see the existing users as noise while the existing users are not affected by the presence of the users who joined the system after them. That is, they can continue to operate exactly as before. The only changes need to be made at the BS. For the uplink, the base station is the decoder and thus the solution hinges on the optimal decoding order, whereas for the downlink the base station is the encoder and the solution is based on the encoding order. Note that as users leave the system the same structure is maintained. As a user exits, it does not affect the rates of the users who joined the system before him. It does help the users who joined the system after him since they no longer have to face interference from his signal.

With multiple cells, we found that the uplink was effectively the same as a single cell system since all the base stations are treated as one composite base station. Thus the single cell strategy extends to multiple cells without loss of optimality. In contrast to the uplink, while the downlink is also viewed as a single virtual base station, there is a refinement since each of the actual base stations has a separate total power constriant. Consequently, the multiple cell downlink solution is different in that the distinct total transmit power constraints require a multi-stage waterfilling solution in determining the optimal input covariance matrix for each user. At each stage, waterfilling is performed until each base station meets its total power constraint. Those base stations that have already met their power constraints are not considered in the successive waterfilling stages.

We also proved a general result that extends beyond our framework. We showed that the achievable rate region with Successive Decoding and timesharing is contained within the achievable rate region with Known Interference Coding and time sharing.

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