Multipath can be described in two domains: time and frequency

Time domain: Impulse response

Frequency domain: Frequency response
Introduction to OFDM modulation

Data coded in frequency domain

Transformation to time domain: each frequency is a sine wave in time, all added up.

Receive

Time-domain signal

Decode each frequency bin separately

Channel frequency response

Transmit

Frequency-domain signal
Introduction to OFDM (Orthogonal frequency division multiplex)

**N carriers**

A user utilizes all carriers simultaneously to transmit its data as coded quantity at each frequency carrier, which can be quadrature-amplitude modulated (QAM).

**Intercarrier Separation** = Any integer Multiple of 1/(symbol duration)

**Features**

- No intercarrier guard bands
- Controlled overlapping of bands
- Maximum spectral efficiency (Nyquist rate)
- Easy implementation using IFFTs
- Very sensitive to time-freq. synchronization

**Modulation technique**

A user utilizes all carriers simultaneously to transmit its data as coded quantity at each frequency carrier, which can be quadrature-amplitude modulated (QAM).
OFDM Modulation and Demodulation using FFTs

Data coded in frequency domain: one symbol at a time

IFFT (Inverse fast Fourier transform)

Data in time domain: one symbol at a time

P/S (Parallel to serial converter)

Transmit time-domain samples of one symbol

d0, d1, d2, …., dN-1

S/P (Serial to parallel converter)

Receive time-domain samples of one symbol

d0’, d1’, d2’, …., dN-1’

FFT (Fast Fourier transform)

Decode each frequency bin independently

b0’, b1’, b2’, …., bN-1’
Loss of orthogonality (by frequency offset)

Transmission pulses
\[ \psi_k(t) = \exp(j k 2 \pi t / T) \]
\[ \psi_{k+m}(t) = \exp(j 2 \pi (k + m) t / T) \]

Reception pulse with offset \( \delta \)
\[ \psi_{k+m}(t) = \exp(j 2 \pi (k + m + \delta) t / T) \quad \text{con} \quad |\delta| \leq 1 / 2 \]

Interference between channels \( k \) and \( k+m \)
\[ I_m(\delta) = \int_0^T \exp(j k 2 \pi t / T) \exp(-j (k + m + \delta) 2 \pi t / T) dt = \frac{T(1 - \exp(-j 2 \pi \delta))}{j 2 \pi (m + \delta)} \]

Summing up
\[ \sum_m I_m^2(\delta) = (T \delta)^2 \sum_{m=1}^{N-1} \frac{1}{m^2} \approx (T \delta)^2 \frac{23}{14} \quad \text{for} \quad N >> 1 \quad (N > 5 \quad \text{is enough}) \]

Loss for 8 carriers

Total ICI due to loss of orthogonality

Practical limit
\[ \delta \text{ assumed r.v.} \]
\[ \text{Gaussian} \quad \sigma = \delta \]
Loss of orthogonality (time)

Let us assume a misadjustment $\tau$

$$X_i = c_0 \int_{-T/2}^{T/2+\tau} \psi_k(t)\psi^*_l(t-\tau)dt + c_1 \int_{-T/2}^{T/2} \psi_k(t)\psi^*_l(t-\tau)dt$$

2 consecutive symbols

Then

$$|X_i| = \begin{cases} 2T \left| \frac{\text{sen} m \pi \frac{\tau}{T}}{m \pi} \right|, & c_0 \neq c_1 \\ 0, & c_0 = c_1 \end{cases}$$

Or approximately, when $\tau \ll T$

$$\frac{|X_i|}{T} \approx \frac{2 \pi \frac{\tau}{T}}{m \pi} = 2 \frac{\tau}{T}$$

Independent on $m$

In average, the interfering power in any carrier is

$$E \left[ \frac{|X_i|^2}{T^2} \right] = 4 \left( \frac{\pi}{T} \right)^2 \frac{1}{2} + 0 \frac{1}{2} = 2 \left( \frac{\tau}{T} \right)^2$$

ICI due to loss of orthogonality

ICI $\approx 20 \log \left( \sqrt{2} \frac{\tau}{T} \right)$, $\tau \ll T$

Per carrier

Loss for 16 carriers

ICI due to loss of orthogonality

assumed a uniform r.v.

Max. practical limit

Zone of interest

Per carrier

Zone of interest
Including a cyclic prefix to each OFDM symbol

To combat the multipath: including time guards between the symbols

CP functions:
- It **accommodates** the decaying transient of the previous symbol
- It **avoids** the initial transient reaches the current symbol
Symplified scheme of an OFDM transceiver

**Transmitter**
- BITS
- CODER
- Slicer
- IFFT
- Slicer
- DAC
- RF
- Cyclic prefix (CP)
- PLL, symbol timing
- Channel estimation
- Frequency offset
- f₀

**Receiver**
- RF
- ADC
- Filter
- Slicer
- FFT
- Slicer
- DECOD
- Bits
- f₀

**Diagram Details**
- **Transmitter**
  - Input: BITS
  - Output: RF
  - Key components: CODER, Slicer, IFFT, DAC, RF, Cyclic prefix (CP)
  - PLL, symbol timing, Channel estimation, Frequency offset

- **Receiver**
  - Input: RF
  - Output: Bits
  - Key components: ADC, Filter, Slicer, FFT, DECOD, Channel estimation, Frequency offset

Windowing of the OFDM symbol

It is interesting to have few carriers as well:

- To introduce short delay in data gathering and signal processing (FFTs)
- To have a bigger intercarrier separation --> It reduces the relative frequency offset

Maintaining a fix bandwidth, if $N$ increases, adjacent channel interference decreases.

BUT

If $N$ increases, the total band used by OFDM also increases. Therefore, some compromise is needed.

Need to shape the OFDM symbols.
OFDM modulators with symbol shaping

Equivalent architectures

\[ p(t) e^{j\omega_{nt}} \]

\[ \sum a_n p(t) e^{j\omega_{nt}} \]

Implemented with FFT

After the synchronous reception

\[ I = \int_{-T/2}^{T/2} p(t) e^{j2\pi(k-n)t/T} dt = 2 \int_{0}^{T/2} p(t) \cos[2\pi(n-k)t/T] dt = \begin{cases} 0, & k \neq n \\ 1, & k = n \end{cases} \]

The simplest way to maintain symmetry within \(-T/2 < t < T/2\) is \(p(t) = k\)

- Symbol shaping has to be carried out as part of the symbol duration + CP
- The total ACI can be considerably reduced
Robustness against the channel and ACI improvement

Virtual OFDM symbols within the slot

- With guards (Cyclic prefixes), the channel’s time dispersion is avoided

\[ L = N + CP \]

OFDM symbols with time guards (CPs)

- With smooth transitions between symbols, the adjacent channel interference is minimized

OFDM symbols with time guards and symbol shaping
Training symbols: 4 us each
- t: 0.8 us, 16 samples
- GI2: 1.6 us, 32 samples
- T: 3.2 us, 64 samples

Data Symbols: 4 us each
- GI: 0.8 us, 16 samples
- OFDM Symbol: 3.2 us, 64 samples

* Only 52 of the 64 carriers are used.
* 4 of the 52 carriers are used for pilot carriers (no data).

Data rate for each 20 Mhz channel:
- 20 Msamples per second.
- 250 Ksymbols per second.
- 48 data carriers per symbol.
- 1/2 or 3/4 convolutional code.
- 1 bit/carrier (BPSK) to 6 bits/carrier (64 QAM).

Overall:

Turbo mode supports 108 Mbps using 40 Mhz channel.
Robustness against errors: random noise and channel-selected errors

Random noise: primarily introduced by thermal and circuit noise.

Channel-selected errors: introduced by magnitude distortion in channel frequency response.

Errors are no longer random. Interleaving is often used to scramble the data bits so that standard error correcting codes can be applied.
• Requires extremely linear power amplifier design.
Adjacent Channel and Alternate Adjacent Channel Rejection

<table>
<thead>
<tr>
<th>Date rate</th>
<th>Minimum Sensitivity</th>
<th>Adjacent Channel Rejection</th>
<th>Alternate Channel rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Mbps</td>
<td>-82 dBm</td>
<td>16 dB</td>
<td>32 dB</td>
</tr>
<tr>
<td>12 Mbps</td>
<td>-79 dBm</td>
<td>13 dB</td>
<td>29 dB</td>
</tr>
<tr>
<td>24 Mbps</td>
<td>-74 dBm</td>
<td>8 dB</td>
<td>24 dB</td>
</tr>
<tr>
<td>36 Mbps</td>
<td>-70 dBm</td>
<td>4 dB</td>
<td>20 dB</td>
</tr>
<tr>
<td>54 Mbps</td>
<td>-65 dBm</td>
<td>0 dB</td>
<td>15 dB</td>
</tr>
</tbody>
</table>

- Requires joint design of the anti-aliasing filter and ADC.